

Economic Analysis of SONET/WDM UPSR and BLSR Ring Networks Using Traffic Grooming

Donghan Kang¹ · Sungsoo Park^{2†}

¹Information Management Team, Samsung Electro-Mechanics, Suwon, 442-743

²Department of Industrial Engineering, Korea Advanced Institute of Science and Technology, Daejeon, 305-701

트래픽 그루밍을 이용한 SONET/WDM 단방향, 양방향 링 네트워크의 경제성 분석

강동한¹ · 박성수²

¹삼성전기 정보경영팀 / ²한국과학기술원 산업공학과

We consider the traffic grooming problem for the design of SONET/WDM(Synchronous Optical Network/Wavelength Division Multiplexing) ring networks. Given a physical network with ring topology and a set of traffic demands between pairs of nodes, we are to obtain a stack of rings with the objective of minimizing the number of ADMs installed at the nodes. This problem arises when a single ring capacity is not large enough to accommodate all the demands. As a solution method, an efficient algorithm based on the branch-and-price approach has been reported in the literature for the problem in which only unidirectional path switched ring (UPSR) was considered. In this study, we suggest integer programming models and the algorithms based on the same approach as the above one, considering two-fiber bidirectional line switched ring(BLSR/2), and BLSR/4 additionally. Using the results, we compare the number of required ADMs for all types of the ring architecture.

Keyword: SONET/WDM ring networks, traffic grooming, branch-and-price algorithm

1. Introduction

Synchronous Optical Network (SONET) or Synchronous Digital Hierarchy (SDH) self-healing rings (SHRs) (Wu, 1992) are widely used for today's optical transport system due to their simple topology and rapid failure restoration.

Suppose we have a physical network with ring topology and traffic demands between some pairs of nodes. In case a single ring is insufficient to serve all the demands due to its limited capacity (e.g. OC-48 = 2.5Gbps), one common approach is to use a stack of rings. A stack of rings is

composed of overlaid SONET rings with the same topology. It can be realized by simple point-to-point Wavelength Division Multiplexing (WDM) (Ramaswami and Sivarajan, 1998) channels between adjacent nodes, which results in a hierarchical SONET/WDM ring network(Ramaswami and Sivarajan, 1998; Wan *et al.*, 2000; Zhang and Qiao, 2000). It is possible due to WDM technology that allows each fiber to support multiple wavelengths. In this ring network, each wavelength corresponds to a SONET ring.

<Figure 1> (Ramaswami and Sivarajan, 1998) shows a configuration of SONET/WDM ring network.

† Corresponding author : Professor Sungsoo Park, Department of Industrial Engineering, KAIST, Daejeon, 305-701, Fax +82-42-869-3110; E-mail sspark@kaist.ac.kr

The key node equipment in the network consists of optical add-drop multiplexers (OADMs) and SONET ADMs. The OADM installed at each node is assumed to be capable of dropping and adding any number of wavelengths. If a wavelength does not carry traffic terminating at a particular node, the OADM may allow the wavelength to optically bypass that node rather than being electronically terminated. Consequently, the number of ADMs can be reduced.

As ADMs are expensive (typically hundreds of thousands of dollars), a fundamental bandwidth management problem is how to route and groom

the traffic demands to minimize the total number of ADMs. There have been many researches on the *traffic grooming* in SONET/WDM ring networks (Arijs and Demeester, 1998; Chiu and Modiano, 2000; Ramaswami and Sivarajan, 1998; Simmons *et al.*, 1999; Sutter *et al.*, 1998; Wan *et al.*, 2000; Zhang and Qiao, 2000). Numerical examples in Ramaswami and Sivarajan (1998) and Simmons *et al.* (1999) show that the number of ADMs can be reduced by traffic grooming procedure. We give a simple example that discriminates the worst grooming from the best. In <Figure 2>, (a) gives the worst grooming resulting

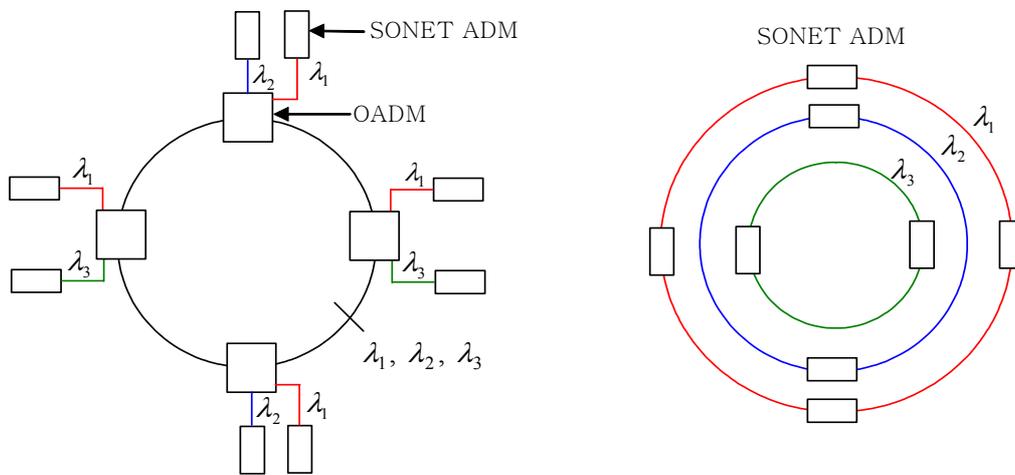
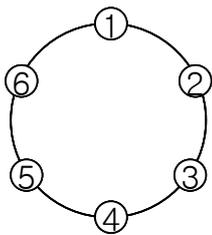


Figure 1. SONET/WDM ring configuration and topology seen by SONET layer.



Node pair	(1,3)	(1,5)	(2,4)	(2,6)	(3,5)	(4,6)
Demand	2	2	2	2	2	2

Ring capacity: 2 (no unit)

Physical network

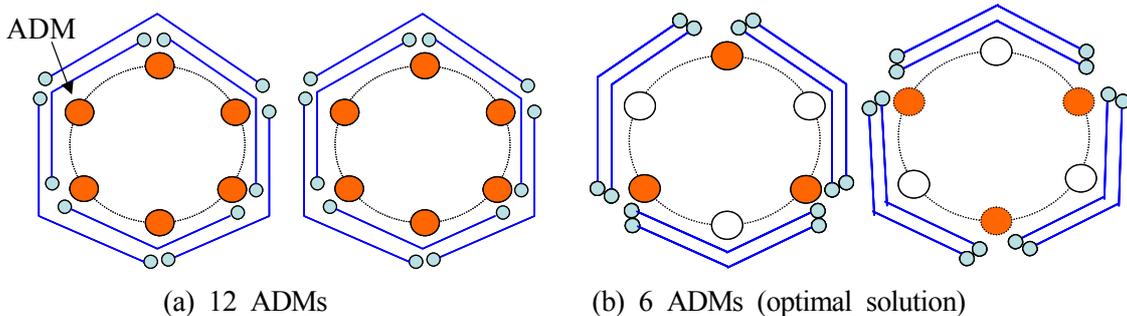


Figure 2. Two extreme cases of traffic grooming.

in 12 ADMs, whereas (b) gives the best grooming resulting in 6 ADMs, where capacity of each ring is two. Please also refer to the numerical example in Ramaswami and Sivarajan (1998).

We assume demand of a node pair can be split by integers and routed in different rings. However, we do not consider the inter-ring traffic. One can consider deploying switches at some nodes to further groom the traffic. Allowing the inter-ring traffic that flows through a switch may result in fewer ADMs but increases complexity in network management (Simmons *et al.*, 1999).

We consider every general architecture of SONET SHRs: unidirectional path switched ring (UPSR), bidirectional line switched ring (BLSR). They both can restore 100% of traffic in case that a fiber cable is cut by rerouting the effected traffic. In a UPSR, working traffic is carried around the ring in one direction. In a BLSR, working traffic travels in both directions over a single path that uses the two parallel communications paths (operating in opposite directions) between nodes of the ring. A BLSR may use two or four fibers depending on the spare capacity arrangement. In a four-fiber BLSR (BLSR/4), a second communications ring, separate from the first, is provided for protection, and working and protection channels use separate communications rings. In a two-fiber BLSR (BLSR/2), working and protection channels use the same fiber with half of the bandwidth reserved for protection. See Wu (1992) for details of SHRs.

In the literature, most previous studies have focused on analysis considering special traffic pattern (such as uniform all-to-all traffic or distance-dependent traffic) or developing heuristic algorithms (Chiu and Modiano, 2000; Ramaswami and Sivarajan, 1998; Simmons *et al.*, 1999; Wan *et al.*, 2000; Zhang and Qiao, 2000). Arijs and Demeester (1998) presented a mathematical formulation of the problem in which BLSR/2s are considered under the condition that routing path of each demand is given in advance. However, simple branch-and-bound gave optimal solutions for only small networks.

Sutter *et al.* (1998) applied the branch-and-price approach to solving the problem in which only UPSR was considered. The algorithm yielded optimal solutions for problem instances of practical size in a short time. However, despite the great concern about the traffic grooming in SONET/WDM ring networks (Arijs and Demeester, 1998; Simmons *et al.*, 1999; Wan *et al.*, 2000; Zhang and Qiao, 2000), to our knowledge, optimization algorithms for the problem considering BLSRs have not been developed yet except just applying

the branch-and-bound procedure. In other words, there has been no research doing the exact economic comparative test between the ring types. Economic comparison is one of the main concern in selecting the proper ring type though they may select it based on another strategy. In this study, we do the exact comparative test by applying the branch and price algorithm (Sutter *et al.*, 1998) for all three ring types and report the number of ADMs needed to route the demand.

UPSR and BLSR/4 use ADMs with TSA (time slot assignment) function, while BLSR/2 uses ADMs with TSI (time slot interchange) function. A node in BLSR/4 is composed of two ADMs or one 1+1 ADM (Wu, 1992). ADMs may cost differently according to their types and vendors. For ease of exposition, we will assume that each node of BLSR/4 in this study is equipped with an 1+1 ADM.

BLSR is an attractive architecture in the sense that it can carry much more traffic than UPSRs of the same capacity though it may result in rather increased system complexity, slower restoration speed, and higher cost of node components for a capacity (Wu, 1992). The capacity requirement is determined by a maximum traffic load over the links of the ring. We assume that integer demand splitting (into two directions along the cycle) is allowed in BLSRs. Note that most previous researches assume that the routing paths are determined in advance (for example, by shortest path routing). We assume the ring capacity is OC-48 and demands are given in STS-1 (51.84Mbps). Thus, in BLSR/2, only 24 STS-1 channels are used as working channel and the other ones are reserved for restoration.

In the next section, we present mathematical formulations of the problem. In section 4, we give a branch-and-price algorithm. We report the computational results in section 5. Section 6 includes the concluding remarks.

2. Formulations

Since a direct formulation (Sutter *et al.*, 1998) of the problem has typically a symmetric structure resulting from the indexing of rings and it leads to well-known difficulties in branch-and-bound, mathematical formulations based on decomposition and column generation are considered in this research; for details of the drawbacks of the direct formulation, refer to Sutter *et al.* (1998). The

formulation of the master problem has an exponential number of columns, but linear programming (LP) relaxation of it can be solved efficiently using column generation technique. Each column corresponds to a feasible ring configuration composed of the ADM nodes and the assigned demands; the objective coefficient is the number of ADMs in it.

Let $G = (V, E)$ be a given physical ring network, where V is the set of nodes and E is the set of links. We assume that the nodes are numbered from 1 to $|V|$ in clockwise direction. Let K be the set of node pairs each of which has a traffic demand between the two nodes. Let R be the set of all feasible ring configurations. The following is the notation used in the formulation of the master problem.

- c_i : the number of ADMs installed in ring configuration $i \in R$.
- d_k : demand of node pair $k \in K$.
- a_{ki} : the amount of demand for node pair $k \in K$ assigned to the ring configuration i .
- x_i : the general integer variable indicating the number of SONET rings (wavelengths) of configuration i .
- L, U : the lower and upper bounds on the number of rings, respectively.

Then the master problem can be formulated as follows.

$$\begin{aligned} \text{Min} \quad & \sum_{i \in R} c_i x_i & (1) \\ \text{s.t.} \quad & \sum_{i \in R} a_{ki} x_i \geq d_k, \quad k \in K & (2) \\ & L \leq \sum_{i \in R} x_i \leq U & (3) \\ & x_i, \text{ nonnegative integer}, \quad i \in R \end{aligned}$$

The objective (1) is to minimize the number of ADMs installed at the nodes. Constraints (2) state that all demands should be satisfied. Constraint (3) specifies the bounds on the number of SONET rings (wavelengths). Typically, $L = 1$ since it will lead to little difficulties in real operation of the network unless other constraints such as the limit on the number of ADM nodes in a ring are imposed.

Column generation procedure is described as follows. Given a restricted formulation of LP relaxation of the master problem with the column set $R' \subset R$, the current optimal solution is also optimal for the LP relaxation of the master problem if all reduced costs for columns

$i \in R \setminus R'$ are at least zero. The reduced cost for a column i is $c_i - \sum_{k \in K} \pi_k^* a_{ki} - \phi_1^* - \phi_2^*$. Here, $\pi_k^* \geq 0$, $\phi_1^* \geq 0$, and $\phi_2^* \leq 0$ are optimal values of the dual variables associated with the k -th constraint in (2), and the two cardinality constraints (3), respectively. Therefore, if any columns with negative reduced costs are found, they are added to the restricted formulation and it is re-optimized. Otherwise, we have solved the LP relaxation of the master problem.

The subproblem is to generate a favorable ring configuration to be added to the formulation as an entering column. The notation used in the formulation of the subproblem for generating an UPSR (SPU) is as follows.

- w_k : the general integer variable indicating the amount of demand of node pair k routed in this ring configuration.
- t_i : the binary variable that is 1 if an ADM is installed at node $i \in V$ and 0 otherwise.
- b : the capacity of SONET rings; $b = 48$ in this research.

Then the SPU can be formulated as follows (Sutter *et al.*, 1998).

$$\begin{aligned} \text{(SPU) Min} \quad & \sum_{i \in V} t_i - \sum_{k \in K} \pi_k^* w_k - \phi_1^* - \phi_2^* & (4) \\ \text{s.t.} \quad & w_k \leq d_k t_i, \quad w_k \leq d_k t_j, \quad k = (i, j) \in K & (5) \\ & \sum_{k \in K} w_k \leq b & (6) \\ & t_i \in \{0, 1\}, \quad i \in V \\ & w_k, \text{ nonnegative integer}, \quad k \in K \end{aligned}$$

The objective function (4) represents the reduced cost of the generated column. Constraints (5) state that up to d_k units of demand can be routed only if ADMs are installed at nodes i and j , where $k = (i, j)$. Constraint (6) represents that the capacity of the UPSR is b .

We use the following notation to formulate subproblem for generating a BLSR/4 (SPB4). Unlike SPU, we should introduce two variables indicating traffic flows into two different directions for each node pair for demand: clockwise and counter-clockwise.

- K_e^{-1} : the set of node pairs for which clockwise paths pass link e .

K_e^2 : the set of node pairs for which counterclockwise paths pass link e .

w_k^1 : the general integer variable indicating the amount of demand for node pair k routed in clockwise direction.

w_k^2 : the general integer variable indicating the amount of demand for node pair k routed in counterclockwise direction.

Then the subproblem can be formulated as follows.

(SPB4) Min

$$\sum_{i \in V} t_i - \sum_{k \in K} \pi_k^* (w_k^1 + w_k^2) - \phi_1^* - \phi_2^*$$

s.t.

$$w_k^1 + w_k^2 \leq d_k t_i, \quad w_k^1 + w_k^2 \leq d_k t_j, \quad (7)$$

$$k = (i, j) \in K$$

$$\sum_{k \in K_1^i} w_k^1 + \sum_{k \in K_2^i} w_k^2 \leq b, \quad e \in E, \quad (8)$$

$$t_i \in \{0, 1\}, \quad i \in V$$

$$w_k^1 \text{ and } w_k^2, \text{ nonnegative integers, } k \in K \quad (9)$$

Constraints (7) state that the demand for node pair k can be split by integers and routed in both directions along the cycle if both nodes have ADMs. Constraints (8) state that traffic load at each link should be within working capacity of BLSR/4 (Wu, 1992). If BLSR/2 is used instead of BLSR/4, the righthand side becomes $b/2$. The corresponding problem is denoted as SPB2.

3. Computational Complexity

The traffic grooming problem for UPSRs is NP-hard (Chiu and Modiano, 2000). NP-hardness for BLSRs can be found in Wan *et al.* (2000). SPU has been proved to be NP-hard (Sutter *et al.*, 1998). We can similarly show that SPB4 is NP-hard by performing pseudo-polynomial transformation from clique problem (Garey and Johnson, 1979).

Clique problem

INSTANCE: A graph $G = (V, E)$ and a positive integer $N \leq |V|$.

QUESTION: Does G contain a clique of size N or more, that is a subset $V' \subseteq V$ such that $|V'| \geq N$ and every two nodes in V' are joined by an edge in E ?

On the other hand, the decision problem version of SPB4 is defined as follows.

SPB4 upper-bound feasibility problem

INSTANCE: A ring network $G = (V, E)$, a set of pairs of nodes K , integer demand d_k for each $j \in K$, a real value $\pi_k^* \geq 0$ for each $k \in K$, ring capacity b , and real values $\phi_1^* \geq 0, \phi_2^* \leq 0, B$.

QUESTION: Is there a subset $V' \subseteq V$ and the allocation of the demand in the two directions w_k^1, w_k^2 for each $k \in K$ such that

$$w_k^1 + w_k^2 \leq \begin{cases} d_k & \text{if } i, j \in V', \text{ where } k = (i, j) \\ 0 & \text{otherwise,} \end{cases}$$

$$\sum_{k \in K_1^i} w_k^1 + \sum_{k \in K_2^i} w_k^2 \leq b, \quad e \in E,$$

$$|V'| - \sum_{k \in K} \pi_k^* (w_k^1 + w_k^2) - \phi_1^* - \phi_2^* \leq B ?$$

Proposition 1. The SPB4 upper-bound feasibility problem is strongly NP-complete.

Proof. The problem is clearly in NP. Let an arbitrary instance of Clique problem be given by the graph $\tilde{G} = (\tilde{V}, \tilde{E})$ and the positive integer $N \leq |\tilde{V}|$. Consider a physical network with ring topology, $G = (V, E)$ such that $V = \tilde{V}$ and E is defined as the set of edges $(1, 2), (2, 3), \dots, (|\tilde{V}| - 1, |\tilde{V}|)$, where nodes in \tilde{V} are numbered arbitrarily. Set $K = \tilde{E}$, $b = N(N-1)/2$, $d_k = 1$, $\pi_k^* = 1$ for $k \in K$, and $\phi_1^* = \phi_2^* = 0$, $B = N - N(N-1)/2$. Then there is a clique of size N or more for \tilde{G} if and only if there is a feasible solution of value no more than $N - N(N-1)/2$. Note that since b is large enough and all demands are 1, any routing of demands does not result in an infeasible solution to the subproblem. Since all numbers in the constructed instance are $O(|\tilde{V}|^2)$, we have given a pseudo-polynomial transformation from Click problem to SPB4 upper-bound feasibility problem.

We can show the NP-hardness of SPB2 by setting $b = N(N-1)$.

4. Branch-and-Price Algorithm

Branch and price, a generalization of branch and bound with LP relaxation, allows column generation to be applied throughout the branch-and-bound tree. This approach has been shown to be very effective to solve huge integer programming prob-

lems that are difficult to solve using other methods. Please refer to Barnhart *et al.* (1998), Vanderbeck (2000) and Vanderbeck and Wolsey (1996) for details of the algorithmic issues and applications. <Figure 3> shows the flow chart of the general branch-and-price algorithm.

4.1 Initial Columns

Initial columns are necessary to start the column generation. First, we set them as a trivial diagonal matrix such that k -th diagonal element is $\min\{d_k, b'\}$ and other components are all zero, where b' is b if the ring architecture is UPSR or BLSR/4 and $b/2$, otherwise. Objective coefficient of each column is 2. Second, we generate other columns through a greedy-style heuristic, which is described next for the case of BLSRs.

We consider an optimization problem to maximize the sum of demands that can be routed in a BLSR/4 of capacity b ($b/2$ in case of BLSR/2). The problem can be formulated as follows. Description of the notation is omitted, since it can easily be inferred from the former one for SBP4.

$$\begin{aligned} \text{Max} \quad & \sum_{k \in K} (w_k^1 + w_k^2) \\ \text{s.t.} \quad & w_k^1 + w_k^2 \leq d_k, \quad k \in K \end{aligned} \quad (10)$$

(8), (9)

Constraints (10) say that up to d_k units of demand k can be routed. To our knowledge, the complexity of this problem has not been reported, but we found after some preliminary tests that it could be solved efficiently by applying a standard branch-and-bound procedure. After we solve the problem to optimality, we may have some of the demands $d_k, k \in K$ still unsatisfied. We then solve the problem again for the unsatisfied demands. We repeat the procedure until all demands are covered. A new column is obtained when we solve the problem once.

4.2 Branching

When LP relaxation of the master problem has been solved to optimality and the solution is not integral, a branching procedure follows. As known well in the literature (Barnhart *et al.*, 1998; Vanderbeck, 2000; Vanderbeck and Wolsey, 1996), the

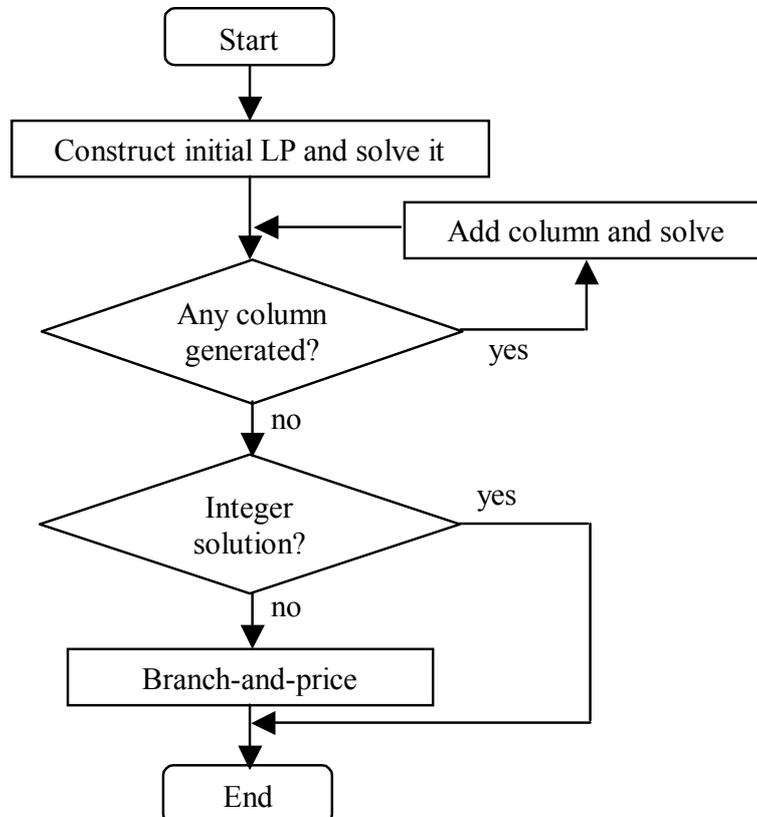


Figure 3. Branch-and-price algorithm.

conventional branching rule based on variable dichotomy may require finding a column of up to n -th lowest reduced cost at depth n in the branch-and-bound tree, which is generally intractable if the subproblem is NP-hard. Instead, we use the branching rule due to Sutter *et al.* (1998) and Vanderbeck(2000). The branching rule is composed of two phases. The first phase determines the assignment of ADM nodes to the stack of rings. In the second phase, we determine the assignment of demands to the stack of rings, which is theoretically indispensable for guaranteeing an optimal solution. Actually, however, the second phase is not necessary, and the first one is sufficient for the purpose.

Phase 1

The first phase is the specific case of the general branching framework in Vanderbeck and Wolsey (1996) or Vanderbeck (2000). We branch on the artificial variables X_{V_2} indicating the number of rings each in which ADMs are installed at all nodes in $V_2 \subseteq V$. Variables are searched in non-decreasing order of $|V_2|$.

Let X^* be an optimal solution to the LP relaxation of the master problem at a node of the branch-and-bound tree. If a value $X_{V_2}^*$ is not integer, two nodes are generated by adding the next inequalities: $X_{V_2} \geq \lfloor X_{V_2}^* \rfloor + 1$ and $X_{V_2} \leq \lfloor X_{V_2}^* \rfloor$, respectively, to the master problem formulation. Note that, actually, X_{V_2} is expressed as the sum of x_i 's such that ADMs are installed at all nodes in V_2 in configuration $i \in R$. For example, let $V_2 = \{1, 2\}$ and suppose that node 1 and node 2 are currently equipped with ADMs in configurations 1, 3, 5 and 8. Suppose also that we have $X_1^* = x_1^* + x_3^* + x_5^* + x_8^* = 1.5$, then we generate two child nodes with additional inequalities (constraints)

$$\begin{aligned} x_1 + x_3 + x_5 + x_8 &\geq 2 \text{ and} \\ x_1 + x_3 + x_5 + x_8 &\leq 1, \end{aligned}$$

respectively.

Suppose we have selected a node of the branch-and-bound tree and solved the LP with the constraint, $X_{V_2} \geq \lfloor X_{V_2}^* \rfloor + 1$. We assume that no more constraints have been added for simplicity. Let $\mu_{V_2}^{1*} \geq 0$ be an optimal dual value associated with the additional constraint. Because of the constraint, the reduced cost for a column index i becomes

$c_i - \sum_{k \in K} \pi_k^* a_{ki} - \phi_1^* - \phi_2^* - \mu_{V_2}^{1*} I_{V_2}^i$, where $I_{V_2}^i = 1$ if all nodes in V_2 are equipped with ADMs in ring configuration i , and 0 otherwise. Let z be a 0-1 variable such that $z = 1$ if and only if $t_i = 1$ for all $i \in V_2$ and 0 otherwise; that is, $z = \prod_{i \in V_2} t_i$. Then the formulation of the subproblem is modified as follows.

Modification of the subproblem formulation

- The term ' $-\mu_{V_2}^{1*} z$ ' is added to the current objective function. For example, the objective function of SPB4 becomes $\sum_{i \in V} t_i - \sum_{k \in K} \pi_k^* (w_k^1 + w_k^2) - \phi_1^* - \phi_2^* - \mu_{V_2}^{1*} z$.
- The next inequalities are added to the current formulation of the subproblem.

$$z \leq t_i \text{ for } i \in V_2 \tag{11}$$

$$z \geq 1 - \sum_{i \in V_2} (1 - t_i) \tag{12}$$

Note that these inequalities are adopted to linearize the equation $z = \prod_{i \in V_2} t_i$.

We can modify the subproblem similarly in case that we solve LP with the constraint, $X_{V_2} \leq \lfloor X_{V_2}^* \rfloor$, and $\mu_{V_2}^{2*} \leq 0$, an optimal dual value associated with the inequality, is obtained. It is noticeable that, since we aim at minimizing the objective function, not all inequalities in (11) and (12) are necessary to solve the modified subproblem. In case $\mu_{V_2}^{1*} > 0$ is used, the inequality (12) is not necessary. On the other hand, when $\mu_{V_2}^{2*} < 0$ is used, the inequalities (11) are not necessary. However, if $\mu_{V_2}^{1*}$ or $\mu_{V_2}^{2*}$ is zero, all of the inequalities are necessary.

The drawback of the above branching rule is that more additional constraints are added to the formulations of the master problem and subproblem as the depth of the branch-and-bound tree increases. In real implementation, however, no branching had occurred such that $|V_2| \geq 3$.

If all the artificial variables are integral, we are given a node-ring assignment (hereafter, node assignment) denoting an assignment of ADMs to several rings of a stack. The node assignment is said to be *feasible* if it can route all the given demands. If it is feasible, we have a feasible solution to the traffic grooming problem, and the incumbent solution (the best solution found so far) may be updated. We can check the feasibility of a node assignment by solving a maximum flow problem for the case of UPSRs (Sutter *et al.*,

1998). For the case of BLSRs, we consider an optimization problem for checking the feasibility. Let R_1 be the set of rings of the stack. To formulate the problem, we introduce decision variables w_{kl}^1 and w_{kl}^2 . The variable w_{kl}^1 (or w_{kl}^2), $k \in K, l \in R_1$ represents the amount of demand k routed clockwise (or counterclockwise) in ring l . Then the problem can be formulated as follows.

$$\begin{aligned}
 \text{Max} \quad & \sum_{l \in R_1} \sum_{k \in K} (w_{kl}^1 + w_{kl}^2) \\
 \text{s.t.} \quad & w_{kl}^1 + w_{kl}^2 \leq d_k I(l, k), \quad k \in K \\
 & \sum_{k \in K_c^1} w_{kl}^1 + \sum_{k \in K_c^2} w_{kl}^2 \leq b, \\
 & e \in E, l \in R_1 \\
 & w_{kl}^1, w_{kl}^2, \text{ nonnegative integers,} \\
 & k \in K, l \in R_1
 \end{aligned} \tag{13}$$

In constraints (13), $I(l, k) = 1$ if nodes i and j are equipped with ADMs in ring l and 0 otherwise, where $k = (i, j)$. The problem can be solved in a short time empirically by applying the branch-and-bound procedure.

If the optimal value is $\sum_{k \in K} d_k$, we are given a new feasible solution to the traffic grooming problem as described earlier. On the other hand, if the node assignment is infeasible, we enter into phase 2.

Phase 2

When an infeasible node assignment is obtained at a branch-and-bound node, we need additional branching based on a set of bounds on the components (a_{ki} 's) of $i \in R$ in order to get an optimal solution; for details of the branching procedure, refer to Vanderbeck (2000). However, it is our experience that such branching rarely (actually never) happens.

4.3 Primal Heuristic

At every tenth node of the tree, we try to get a feasible solution using a variable-fixing heuristic. The variable of the highest value is fixed to the least integer greater than or equal to the value, and the linear program is re-optimized. The process is repeated until an integer solution is obtained or it is identified that a feasible solution cannot be obtained. When a feasible solution has been obtained, the incumbent solution may be updated. We did not generate any more columns after fixing some variables because it increased running time much.

5. Computational Results

We generated random data and coded a computer program in C to test the branch-and-price algorithm. The generated data can be classified into four groups according to node-'node pair' sizes, namely, (7, 12), (10, 18), (13, 31), and (15, 31), where node pair size refers to the number of pairs of nodes between which demand is defined. The sizes were adopted considering practical planning scenarios in Korea and they are not less than those shown in the literature (Sherali *et al.*, 2000; Sutter *et al.*, 1998). Demand values were generated as follows: 90% of demands were generated uniformly between 1 and 10, while the other 10% of them were generated uniformly between 11 and 25. We generated three test problems for each node-'node pair' size.

<Table 1> summarizes the results of the tests on Intel Pentium III PC with CPU of 866MHz. Heading 'Opt' refers to the optimal objective value of the problem. Gap represents relative ratio between IP and LP , and defined as $(IP - LP)/IP \times 100 (\%)$, where IP and LP are the optimal objective values of the problem and LP relaxation of it, respectively. '#Rings' refers to the number of SONET rings (wavelengths used). '#B&B nodes' refers to the number of nodes generated in the branch-and-price algorithm. '#Columns' refers to the number of generated columns (including initial columns). Finally, 'Time(s)' refers to running time in seconds. We used callable library of ILOG CPLEX 7.0 as LP and MIP solvers.

We could obtain optimal solutions for all test problems in a reasonable time. Percentage gaps range from 0% to 17%. For all test problems, we set the bounds on the number of wavelengths as $L = 1$ and $U = 7$. One can set them to any values such that $L \leq U$, but careless setting may make the problem infeasible. Some test problems such as the second one with size (7, 12) result in a stack of one ring when the ring architecture is BLSR/4, which means a ring is sufficient to route all demands.

We also consider data of another special demand types. Problem size (n, n) means that pairs of adjacent nodes have demands, that is, $K = \{(1, 2), (2, 3), \dots, (n-1, n)\}$, and $d_k = 30$, $k \in K$, where $n = 7, 10, 13, 15$. Also, size $(n, n-1)$ means that demands are given between one hub and the remaining nodes, that is, $K = \{(1, 2), (1, 3), \dots, (1, n)\}$, and $d_k = 30$,

Table 1. Computational results

a) UPSR

Problems		Opt.	Gap (%)	#Rings	#B&B nodes	#Columns	Time (s)
(7, 12)	1	12	11	3	14	77	1.1
	2	11	5	3	0	48	0.4
	3	11	9	3	6	66	0.8
(10, 18)	1	16	12	3	17	193	4.9
	2	16	7	3	9	115	2.3
	3	16	5	3	14	139	3.4
(13, 31)	1	25	8	5	35	466	31.6
	2	27	4	6	29	318	24
	3	22	4	4	20	413	25.4
(15, 31)	1	27	4	6	14	215	11.9
	2	26	7	5	140	981	87.1
	3	25	5	5	84	749	68.8

b) BLSR/2

Problems		Opt.	Gap (%)	#Rings	#B&B nodes	#Columns	Time (s)
(7, 12)	1	12	17	3	9	88	1.7
	2	11	17	3	5	88	1.6
	3	10	5	2	0	52	0.9
(10, 18)	1	14	5	2	6	174	7.7
	2	16	12	4	34	203	10.1
	3	16	9	3	9	117	5.8
(13, 31)	1	22	7	4	75	1258	219.7
	2	24	6	5	93	1146	222.2
	3	21	8	4	34	461	55.6
(15, 31)	1	26	5	6	24	314	54
	2	24	7	4	133	1541	318.2
	3	24	8	4	65	645	113.9

c) BLSR/4

Problems		Opt.	Gap (%)	#Rings	#B&B nodes	#Columns	Time (s)
(7, 12)	1	9	15	2	2	65	0.5
	2	7	0	1	0	45	0.1
	3	7	0	1	0	57	0.1
(10, 18)	1	10	0	1	0	123	0.4
	2	12	8	2	4	131	3.3
	3	12	9	2	6	157	4.3
(13, 31)	1	17	6	2	14	732	171.9
	2	18	5	2	12	433	59.3
	3	17	10	2	5	337	30.5
(15, 31)	1	21	7	3	8	409	52.8

Table 2. Computational results: demands of special type

a) demands between adjacent nodes

Problems	Ring type	Opt.	Gap (%)	#Rings	#B&B nodes	#Columns	Time (s)
(7, 7)	U	14	6	5	16	28	0.5
(10, 10)	U	20	6	7	27	51	1
(13, 13)	U	Inf.	-	-	-	-	-
(15, 15)	U	Inf.	-	-	-	-	-
(7, 7)	B2	13	33	2	33	68	1
(10, 10)	B2	19	34	6	68	114	3.4
(13, 13)	B2	25	35	7	83	166	7.6
(15, 15)	B2	29	35	7	670	1032	79.8
(7, 7)	B4	7	0	1	0	15	0
(10, 10)	B4	10	0	1	0	31	0.1
(13, 13)	B4	13	0	1	0	30	0.1
(15, 15)	B4	15	0	1	0	56	0.1

b) demands between one hub and the remaining nodes

Problems	Ring type	Opt.	Gap (%)	#Rings	#B&B nodes	#Columns	Time (s)
(7, 6)	U	12	6	5	10	32	0.4
(10, 9)	U	18	6	7	14	54	1.1
(13, 12)	U	Inf.	-	-	-	-	-
(15, 14)	U	Inf.	-	-	-	-	-
(7, 6)	B2	12	6	5	10	41	1
(10, 9)	B2	18	6	6	30	78	0.5
(13, 12)	B2	Inf.	-	-	-	-	-
(15, 14)	B2	Inf.	-	-	-	-	-
(7, 6)	B4	8	0	2	0	16	0.1
(10, 9)	B4	12	0	3	5	37	1
(13, 12)	B4	16	0	4	6	52	2.9
(15, 14)	B4	19	2	5	149	217	8.6

Inf : Infeasible, U : UPSR, B2 : BLSR/2, B4 : BLSR/4

$k \in K$, where node 1 is the hub. Computational results on the data are reported in <Table 2>. We can see that all problems can be solvable in a short time, but the limited number of wavelengths ($= 7$) causes some test problems like (13, 13) with UPSR to be infeasible. The limit is more restrictive with the architecture UPSR than with BLSR/2 or BLSR/4; it is most unrestrictive with BLSR/4, which can be easily inferred from the observation that it can hold the largest amount of demands given a capacity. All problems of size (n, n) result in one ring for BLSR/4 and the

optimal values are n since $d_k < 48$. Clearly, we have the same result for BLSR/2 if $d_k \leq 24$.

<Table 3> shows the (average) ratio of the number of ADMs for BLSRs to the number for UPSR. We can see that the number of required ADMs for BLSR/2 is equal to or slightly less than that for UPSR (on average). The relative number of ADMs for UPSR : BLSR/2 : BLSR/4 is 1 : 0.9-1 : 0.5-0.8, where the relative number for UPSR is 1. Note, however, that the ratios may be changed depending on the problem instances — please recall that both BLSR/2 and

BLSR/4 can route all demands with n ADMs for the instances (n, n) with $d_k \leq 24$. The table also helps us make a decision on the selection of the appropriate ring architecture in terms of total node (ADM) cost. For example, if the node cost of BLSR/4 is 1.3 times higher than that of UPSR for the problems of size $(7, 12)$, we can conclude that using BLSR/4 is more economical than using UPSR.

Table 3. (Average) Ratio of the number of ADMs for BLSR to the number for UPSR

Problem size	B2 / U	B4 / U
(7, 12)	0.97	0.67
(10, 18)	0.96	0.71
(13, 31)	0.91	0.71
(15, 31)	0.95	0.76
(7, 7)	0.93	0.5
(10, 10)	0.95	0.5
(13, 13)	-	-
(15, 15)	-	-
(7, 6)	1	0.67
(10, 9)	1	0.67
(13, 12)	-	-
(15, 14)	-	-

U : UPSR, B2 : BLSR/2, B4 : BLSR/4

6. Concluding Remarks

In this paper, we developed an efficient traffic grooming algorithm for the design of SONET/WDM ring networks. For each type of the BLSR, we presented integer programming models of the problem and developed an efficient algorithm based on the branch-and-price approach. We showed that our algorithms could solve the problem instances of practical size to optimality within a reasonable time. Using the result, we compared the number of required ADMs for all types of the ring architectures.

In this study, we set the bounds on the number of SONET rings in the master formulation. However, there is another way of controlling the number of SONET rings: introducing a fixed cost of a ring. The value can be set by a planner. As the cost of a ring becomes higher, smaller number

of rings will be generated. No changes are made in the branch-and-price algorithm except in the objective functions of the formulations of the master problem and subproblem, respectively.

As described in section 1, if some nodes are equipped with switches for the inter-ring transmission, the number of ADM nodes may be reduced further. However, the decomposition-based model in this study cannot be applied in that case. Some other problem-specific methods would be needed to solve the problem.

References

- Arijs, P., and Demeester, P. (1998), Efficient Design of Stacked Multiplex Section Shared Protection Ring. In: D.W. Faulkner and A.L. Harmer, eds., *Long-Haul, ATM and Multi-media Networks*, IOS Press, Amsterdam, The Netherlands, 164-171.
- Barnhart, C., Johnson, E.L., Nemhauser, G.L., Savelsbergh, M.W.P., and Vance, P.H. (1998), Branch-and-price: Column Generation for Solving Huge Integer Programs, *Operations Research*, **46**, 316-329.
- Chiu, A.L., and Modiano, E.H. (2000), Traffic Grooming Algorithms for Reducing Electronic Multiplexing Costs in WDM Ring Networks", *IEEE Journal of Lightwave Technology*, **18**, 2-12.
- Garey, M.R., and Johnson, D.S. (1979), *Computer and Intractability*, Freeman, San Francisco.
- Ramaswami, R., and Sivarajan, K. (1998), *Optical Networks: A Practical Perspective*, Morgan Kaufman, San Francisco, CA.
- Sherali, H.D., Smith, J.C. and Lee, Y. (2000), Enhanced Model Representations for an Intra-ring Synchronous Optical Network Design Problem Allowing Demand Splitting, *INFORMS Journal on Computing*, **12**, 284-298.
- Simmons, J.M., Goldstein, E.L. and Saleh, A.A.M. (1999), Quantifying the Benefit of Wavelength Add-drop in WDM Rings with Distance-independent and Dependent Traffic, *IEEE Journal of Lightwave Technology*, **17**, 48-57.
- Sutter, A., Vanderbeck, F., and Wolsey, L. (1998), Optimal Placement of Add/drop Multiplexers: Heuristic and Exact Algorithms, *Operations Research*, **46**, 719-728.
- Vanderbeck, F. (2000), On Dantzig-Wolfe Decomposition in Integer Programming and Ways to Perform Branching in a Branch-and-price Algorithm, *Operations Research*, **48**, 111-128.
- Vanderbeck, F., and Wolsey, L., (1996), An Exact Algorithm for IP Column Generation, *Operations Research Letters*, **19**, 151-159.
- Wan, P.-J., Călinescu, G., Liu, L. and Frieder, O. (2000), Grooming of Arbitrary Traffic in SONET/WDM BLSRs, *IEEE Journal on Selected Areas in Communications*, **18**, 1995-2003.
- Wu, T-H. (1992), *Fiber Network Service Survivability*, Artec House, Boston, MA.
- Zhang, X., and Qiao, C. (2000), An Effective and Comprehensive Approach for Traffic Grooming and Wavelength Assignment in SONET/WDM Rings, *IEEE/ACM Transaction on Networking*, **8** (5), 608-617.