Abstract

The Korea Post uses a specific transport equipment called ‘pallet’ to carry mail and packages efficiently. Since some regions send more goods than they receive and others receive more than they send, there exists imbalance between demand and supply for the empty pallets. They should be repositioned to satisfy demands. Empty pallet allocation (EPA) problem involves reallocating empty pallets to cope with this situation.

We analyzed the current EPA process of the Korea Post, in which decisions are still made manually. It can be modeled as the single commodity network flow problem which can be solved in a polynomial time. These models provide us with optimal repositioning schedules of the empty pallets.

1. Introduction

In the Korean postal service, specific transport equipments, pallets are used to carry mail. The pallet makes it easier to carry mail in the sense that it has wheels and serves several postal matters together. For delivery, an empty pallet is filled with mail and sent from one post office to another. The mail is unloaded and the pallet becomes empty at the destination. And then, the empty pallet can be reused. Everyday each office needs empty pallets to carry and sort mail. The necessary empty pallets are regarded as demand. Each office also receives loaded pallets. After unloading, the received loaded pallets become supply for the demand. Unfortunately the supply and the demand are usually out of balance in most offices, and some offices have an excess of empty pallets, while others are short of those. Therefore, they should be repositioned from surplus offices to deficit offices. In this situation, the problem of reallocating empty pallets is called the Empty Pallet Allocation (EPA) problem.

To our knowledge, there have been no published researches on the EPA. However, there are some similar studies on repositioning empty transport equipments to solve imbalance of supply and demand, such as the Empty Freight Distribution (EFD) problems in the land transportation system and the Empty Container Allocation (ECA) problems in the marine time transportation system.

These literatures are categorized according to whether input parameters are uncertain or not, the number of transport equipment types, the length of planning horizon, and the scope of study. When the type of transport equipments is single, the problem becomes the single commodity network flow problem. When there are a number of equipment types, the problem is the multi-commodity network flow problem.


1) This paper is supported by Electronics and Telecommunications Research Institute (ETRI) in 2006.
tardiness as well as transportation cost and dissatisfaction of demand.

There are some studies considering vehicle routing, scheduling and empty transport equipment repositioning together. Powell and Carvalho (1998a, 1998b) developed a logistics queuing network (LQN) model for single commodity and multi-commodity in multi-period. Many real-world details, such as trucks, tasks, time window, etc., are considered. Smilowitz (2006) treated a single period single commodity network flow problem with truck routing and scheduling, but this is more similar to vehicle routing problem (VRP).

In a single commodity case, some literatures handle uncertain input parameters in demand and supply data (see Table 2). Erera et al. (2006) used robust optimization to minimize expected cost. Crainic et al. (1993) proposed a mathematical formulation, 2-T stage stochastic network model, and Cheung and Chen (1998) solved this model by stochastic quasi-gradient method and a hybrid approximation procedure.

The EFD and ECA are related to the EPA, but transportation structure and operation conditions of the Korea Post are different from those of the land system or the marine time transportation system. Since the Korea Post has many offices and available transport routings, its transportation structure is more complex. In addition, it is impossible to forecast demand or supply of empty pallets for a long period, because the supply and demand are decided by fluctuant daily mail and packages, the number of pallets is larger, and the size of those is smaller than freights or containers. Therefore, the studies suitable to the EPA are required. The goal of this paper is to analyze the current EPA process of the Korea Post and to propose models which provide optimal repositioning schedules.

The paper is organized as follows. In section 2, the current EPA process is analyzed. Section 3 is dedicated to the mathematical formulations of the EPA process. Finally, Section 4 discusses the consequences of our study at the Korea Post.

2. Problem Description

In this section, we describe the current Korea Post environment for the EPA. First, the kinds of transportation equipments, the relationship of transportation offices and the guide for managing pallets are represented. Finally, we analyze the current EPA process.

2.1 Empty Pallet Management

The Korea Post uses pallets, trolleys, mail boxes, and mail bags to deliver mail and packages. Among these, only pallets and trolleys have wheels, and they are utilized not for lading but for immediate conveyance. Pallets are the most expensive and biggest units, and they are used for transportation or loading letters, parcels, mail boxes, and mail bags. Due to the recent increase in packages, demands for empty pallets have risen. Thus, efficient empty pallet management is necessary to satisfy these demands.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Period</th>
<th>Scope</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>Single-period</td>
<td>Pure</td>
<td>Crainic et al. (1993)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mixed</td>
<td>Smilowitz (2006)</td>
</tr>
<tr>
<td></td>
<td>Multi-period</td>
<td>Pure</td>
<td>Shan &amp; Khoong (1995)</td>
</tr>
<tr>
<td>Multi-commodity</td>
<td>Single-period</td>
<td>Pure</td>
<td>Crainic et al. (1993)</td>
</tr>
<tr>
<td></td>
<td>Multi-period</td>
<td>Pure</td>
<td>Speieckermann&amp;VoB (1995)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mixed</td>
<td>Holmberg et al. (1998)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Jobern et al. (2004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Powell &amp; Carvalho (1998b)</td>
</tr>
</tbody>
</table>

Table 1 Deterministic Model

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Period</th>
<th>Scope</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>Multi-period</td>
<td>Pure</td>
<td>Crainic et al. (1993)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cheung and Chen (1998)</td>
</tr>
<tr>
<td>Multi-commodity</td>
<td></td>
<td></td>
<td>Erera et al. (2006)</td>
</tr>
</tbody>
</table>

Table 2 Stochastic Model

The Korea Post has the transportation system based on Hub-and-Spoke network. Figure 1 shows the network. There are one Super Management Office (SMO), 27 general Management Offices (MO), and 3699 general Post Offices (PO). According to the transportation system, the process is divided into two parts; the main and the substructure. The main and the substructure consist of the SMO and MOs, and one MO and its subordinate POs respectively. Each MO is connected to all other MOs and the SMO in the land transportation system. In the case of POs, there are not all direct links between each pair of POs, and they links via their super ordinate MO.
Empty pallet transportation also follows this system.

The amount of surplus or necessary empty pallets is decided everyday in comparison with the 'Standing Fixed Number (SFN)'. The SFN is the number of empty pallets that each office should hold for carrying and sorting mail. It is desirable to keep the level of inventory as close as possible to the SFN at the end of everyday. For the sake of this, each office deliberates on inventory, its SFN and the forecast quantity of receiving or sending mail, and then returns or requests empty pallets to its superordinate office in advance.

The SFN for each office is reestimated every six months. And also, they are adjusted upward when the amount of mail is expected to sharply increase. These periods, such as big holidays and the year-end, are called the 'Special Delivery Period (SDP)'. To satisfy the additional demand, empty pallets are released from SMO's reserved inventory. The temporary SFN is introduced several days before the SDP and withdrawn after the SDP.

2.2 The Current EPA Process

Figure 2 shows the current empty pallet allocation process. The main-structure of the EPA is as follows. Before 7 am in the morning, loaded pallets arrive at each MO from other MOs by scheduled trucks. In addition, laden pallets are delivered to its subordinate POs at around 8 am, and received from them by scheduled gathering trucks two times in the afternoon. Loaded pallets are exchanged from 6 pm till 7 am of the next day. Considering above conditions, each MO informs the SMO how many empty pallets are needed more or leftover. The SMO manually commands to transport empty pallets directly from surplus offices to deficit offices by trucks. In an unavoidable case, empty pallets are released from the SMO's reserved inventory.

The EPA of substructure is conducted by each MO. At around 9 am, Mail is carried from the MO to its subordinate POs, and distributed to mailmen for final delivery. Each PO receives mail from customers during business hours. This mail is loaded on pallets and is sent to its superior MO by scheduled trucks twice. The MO grasps the deficit or surplus in empty pallets of its POs, and delivers and collects them by the scheduled trucks.

3. Mathematical model

In this section, EPA models are presented considering the situation described in the prior section.

Since the EPA process is composed of the main and the substructure, two distinct models are needed. However, a MO is influenced by overall empty pallet surplus or deficit of its subordinate POs, so the result of substructure model should be included in the main-structure model to reflect this relationship.

Since pallet transportation is accomplished within a day and the EPA models have little 'end-of-horizon effect', we present models for single day (Choong et al. 2002). However, the SFNs dramatically change on the confines of the SDP. The SFNs increase at the beginning of the SDP, and they decrease at the end of the SDP. To fit the amount of empty pallets for these changes, multi-day EPA model is also required.

Empty pallets are carried by trucks. Among the MOs and SMO, only direct links are used to transport them, but our main-structure model also considers scheduled trucks which ship loaded pallets. Since these scheduled trucks are already settled, the use of them may reduce transportation costs. In substructure, empty pallets are delivered or collected by mail gathering scheduled trucks twice a day. The schedule can not be taken into account manually. Therefore, even if there are POs with surplus empty pallets, MO alone supplies all deficit empty pallets and gathers all surplus empty pallets of POs. In our substructure model, empty pallet transportation among POs is allowed. With this, unnecessary movements of empty pallets decrease.

3.1 Model Description

Each model is based on time-space network. Figure 3 illustrates single day models of main and substructure.
as networks. Each planning horizon is single day and consists of two periods. All nodes, except for two dummy nodes, represent transportation offices (PMO, MOs and POs) and are duplicated in each period. An outflow from a dummy node implies empty pallets which are released from superior office’s reserved inventory. Outflows from the other describe inevitable deficits of empty pallets.

The flow of these networks means empty pallet holding inventory and movements from one office to another. There are four types of arcs; inventory holding, empty pallet transportation, unavoidable demand shortage, and superior office’s supply. The inventory arcs are divided into ‘SFN’ arcs and surplus inventory arcs, and connect offices to the copied offices in next period. Transportation arcs of substructure imply the conveyance just by scheduled trucks, but in main-structure direct unscheduled trucks are included.

We assume that supplies and demands of empty pallets by laden pallets are realized at the beginning of first period, and at the beginning of second period respectively. Scheduled truck capacity is residual after loading, and direct unscheduled truck capacity is the maximum available capacity.

In Multi-day model, single day models above are expanded into a multi-period with more than two.

### 3.2 Model Formulation

Our models are composed of main and substructure model, which cope with the EPA daily and on the confines of the SDP. Usually the substructure model is solved 27 times a day every morning, and with the results the main-structure model is executed once a day. However, main-structure model is solved once a planning horizon on the confines of the SDP.

#### Substructure Model (Single Day) of MO

**NOTATION**

- $C$ := Set of MOs.
- $N^e$ := Set of subordinate POs of $c$ and $e$, $e \in C$.
- $T$ := Set of the gathering times, $T = \{1, 2\}$.
- $A^{st}$ := Set of the available truck schedule in $t$th gathering, $t \in T$.
- $R^{st}_{pq}$ := Set of the available routings included $(p,q)$ pair in $t$th gathering, $\forall (p,q) \in A^{st}$, $t \in T$, $p,q \in N^e$.

**MODEL PARAMETERS**

- $Q_n$ := Initial inventory at $n$, $\forall n \in N \setminus \{c\}$.
- $S_n$ := Supply of empty pallets at $n$, $\forall n \in N \setminus \{c\}$, or amount of receiving mail at $n$, $\forall n \in N \setminus \{c\}$.
- $D_n$ := Demand for empty pallets at $n$, $\forall n \in N \setminus \{c\}$, or amount of sending mail at $n$, $\forall n \in N \setminus \{c\}$.
- $U^{sr}_{pq}$ := Capacity(residual) of the available truck routings, $\forall r \in R^{st}_{pq}$, $(p,q) \in A^{st}$, $t \in T$, $p,q \in N^e$.
- $SFN_n$ := ‘Standing Fixed Number’ at $n$, $\forall n \in N \setminus \{c\}$.
- $P$ := $|N|$.

**DECISION VARIABLES**

- $x^{sr}_{pq}$ := Amount of empty pallets transported from $p$ to $q$ by $r$ in $t$th gathering, $\forall r \in R^{st}_{pq}$, $(p,q) \in A^{st}$, $t \in T$, $p,q \in N^e$.
- $i^1_n$ := Inventory at $n$ after the first gathering, $\forall n \in N^e$.
- $i^2_n$ := SFN at $n$, $\forall n \in N \setminus \{c\}$.
- $i^t_e$ := Inventory at $e$ during the $t$th gathering, $\forall t \in T$.
- $o_n$ := Empty pallet surplus over the SFN at $n$, $\forall n \in N \setminus \{c\}$.
- $e_n$ := Inevitable empty pallet deficit at $n$, $\forall n \in N \setminus \{c\}$.
- $e$ := Amount of empty pallet return from POs to their superior $c$.
- $d$ := Amount of empty pallet request from POs to their superior $c$.

**OBJECTIVE**

$$\text{Min} \sum_{(p,q) \in A^{st}} \sum_{r \in R^{st}_{pq}} x^{sr}_{pq} + 3P \sum_{n \in N \setminus \{c\}} e_n + P \sum_{n \in N \setminus \{c\}} o_n + Pd$$ (1.1)
CONSTRAINS

\[ \begin{align*}
& \sum_{(q,n) \in A^1} x_{q,n}^{1r} - \sum_{(q,n) \in A^1} x_{q,n}^{1r} = S_n + Q_n \quad \forall n \in N \backslash \{c\} \quad (1.2) \\
& y_{n}^{1} = \alpha_{n} + \sum_{(q,n) \in A^1} x_{q,n}^{2r} - \sum_{q \in N} x_{q,n}^{2r} = -D_n \quad \forall n \in N \backslash \{c\} \quad (1.3) \\
& \sum_{(q,n) \in A^1} x_{q,n}^{3r} - d_n = 0 \quad (1.4) \\
& \sum_{(q,n) \in A^1} x_{q,n}^{2r} - \sum_{(q,n) \in A^1} x_{q,n}^{1r} = -D_n \quad \forall n \in N \backslash \{c\} \quad (1.5) \\
& \sum_{(q,n) \in A^1} x_{q,n}^{3r} - s_n = 0 \quad (1.6) \\
& \sum_{(q,n) \in A^1} x_{q,n}^{2r} - s_n = -D_n \quad \forall n \in N \backslash \{c\} \quad (1.7) \\
& 0 \leq x_{p,q} \leq U_{p,q} \quad \forall r \in R_{p,q}^{d}, (p,q) \in A^1, t \in T \quad (1.8) \\
& \text{All variables are integer.} 
\end{align*} \]

The objective of this model is to minimize movements and deficits of empty pallets. The objective function (1.1) is composed of transfer quantity, inevitable shortage, surplus inventory and supply from their superior MO of empty pallets. The third and fourth term represent guiding polices for supply from their superior MO of empty pallets. The objective function (1.1) is composed of transfer to estimated delivery quantity.

Restrictions (1.2) and (1.3) stand for inventory balances at POs. Relations (1.2) state that the amount of inventory at each office after the first gathering plus sending empty pallets, and minus receiving empty pallets by the first gathering trucks is equal to supply by loaded pallets plus initial inventory. In the equations (1.3), they are represented that the number of inventory after the second gathering plus sending empty pallets, minus receiving empty pallets by the second gathering trucks and former inventory is equal to estimated delivery quantity. Restrictions (1.4), (1.5) and (1.6) are about flow balances in MO. However, since deliveries between MOs are not considered in the substructure model, these right-hand sides are zero. They cover the amount of empty pallet return or request from POs to their superior MO.

Third term in objective function and relations (1.7) make inventory to close the SFN at the end of day. The capacity constraints of truck routings are equations (1.8). This model is a network flow model with capacity constraints. Therefore, this can be solved in a polynomial time.

Main-structure Model (Single and Multi-day) NOTATION

\[
\begin{align*}
N & := \text{Set of SMO and MOs, } c := \text{SMO.} \\
T & := \text{Set of days of planning horizon.} \\
A^i & := \text{Set of the available truck schedule in } t, \forall t \in T. \\
R_{p,q}^{d} & := \text{Set of the available routings included (p,q) pair in } t, \forall (p,q) \in A^i. \\
Q_n & := \text{Initial inventory at } n, \forall n \in N. \\
S_n & := \text{Supply of empty pallets at } n \text{ in } t, \forall n \in N, t \in T. \\
D_n & := \text{Demand for empty pallets at } n \text{ in } t, \forall n \in N, t \in T. \\
s_n & := \text{Amount of empty pallet return of subordinate POs at } n \text{ in } t, \forall n \in N. \\
d_n & := \text{Amount of empty pallet request of subordinate POs at } n \text{ in } t, \forall n \in N, t \in T. \\
SUBB_{i}^{p,q} & := \max\{d_{i}^{p,q} - \max\{s_{i}^{p,q}, 0\}, \text{ or Amount of empty pallet return (request) from subordinate POs at } n \text{ in } t, \forall n \in N, t \in T. \\
W_{m} & := \text{Capacity of the unscheduled direct truck from } n \text{ to } m, \forall n, m \in N, n \neq m. \\
SFN & := \text{‘Standing Fixed Number’ at } n \text{ in } t, \forall n \in N, t \in T. \\
L & := \text{Maximum empty pallets that SMO can release from reserved inventory.} \\
M & := \sum_{(p,q) \in A^1} |R_{p,q}^{d}|. \\
\end{align*} \]

DEcision Variables

\[
\begin{align*}
& x_{p,q}^{1r} := \text{Amount of empty pallets transported from } p \text{ to } q \text{ in } t \text{ by } r, \forall r \in R_{p,q}^{d}, (p,q) \in A^i, t \in T, p, q \in N. \\
& y_{n}^{1} := \text{Amount of empty pallets transported from } n \text{ to } m \text{ in } t \text{ by unscheduled direct trucks, } \forall n, m \in N, n \neq m, t \in T. \\
& i_{n}^{1} := \text{Inventory at } n \text{ after the unscheduled direct truck departure at } n \text{ in } t, \forall n \in N, t \in T. \\
& d_{n}^{1} := \text{SFN at } n \text{ in } t, \forall n \in N, t \in T. \\
& o_{n}^{1} := \text{Empty pallet surplus over the SFN at } n \text{ in } t, \forall n \in N, t \in T. \\
& e_{n}^{1} := \text{Inevitable empty pallet deficit at } n \text{ in } t, \forall n \in N, t \in T. \\
& l := \text{The number of empties that the SMO must to release from reserved inventory.} 
\end{align*} \]
The objective of this model is similar to prior submodel structure. However, the objective function (2.1) is a little different. Transfer quantity is divided into the first and second term which represent (2.1) is a little different. Transfer quantity is divided into the first and second term which represent (2.2) to (2.4) stand for flow routings and unscheduled trucks are equations (2.7) and (2.8). Relation (2.9) represents the maximum number of empty pallets which the SMO can release about in first period. Restrictions about inventory balances during scheduled truck operation are equations (2.5). The EPA process of the Korean postal service can be modeled as the single commodity network flow problem which can be solved in a polynomial time.

4. Conclusion

The EPA process of the Korean postal service can be modeled as the single commodity network flow problem which can be solved in a polynomial time. Computational experiments will be carried out on real data. Empty trolleys and pallets can be considered together for the further research.

Our models are expected to be applied to the transportation equipment management at the Korea Post, and it will provide optimal repositioning schedules of the empty pallets.

References


The objective of this model is similar to prior submodel structure. However, the objective function (2.1) is a little different. Transfer quantity is divided into the first and second term which represent

\[ \min \sum_{T} \sum_{(p, q) \in A} x_{p, q} + M \sum_{T} \sum_{n \in N} y_{n,m} \]

\[ + 3MT \left| \sum_{T} \sum_{n \in N} e_{n}^T + MT \left| \sum_{n \in N} e_{n}^T + MT \right| l \] \tag{2.1}

\[ \text{CONSTRAINTS} \]

\[ t_{n,m} - l = Q \] \tag{2.2}

\[ g_{n,m} = Q \] \tag{2.3}

\[ \text{Relations from (2.2) to (2.4) stand for flow} \]

\[ \sum_{q \in N} x_{q,n} - \sum_{q \in N} x_{q,n} = 0 \] \tag{2.4}

\[ e_{n,m} = -D_{n} - \text{SUB}_{n} \] \tag{2.5}

\[ 0 \leq x_{p, q} \leq U_{p, q} \] \tag{2.6}

\[ 0 \leq y_{n,m} \leq W_{n,m} \] \tag{2.7}

\[ l \leq L \] \tag{2.8}

\[ e_{n,m} \geq 0 \] \tag{2.9}

\[ \text{all variables are integer} \]

The objective of this model is similar to prior submodel structure. However, the objective function (2.1) is a little different. Transfer quantity is divided into the first and second term which represent conveyance by scheduled trucks and by unscheduled trucks respectively.

Relations from (2.2) to (2.4) stand for flow balances in MOs and SMO during unscheduled truck operation. Among these, equations (2.2) and (2.3) are about in first period. Restrictions about inventory balances during scheduled truck operation are equations (2.5).

Set of constraints (2.6) and fourth term of objective function keep the level of inventory as close as possible to the SFN at the end of planning horizon. The capacity constraints of scheduled truck routings and unscheduled trucks are equations (2.7) and (2.8). Relation (2.9) represents the maximum number of empty pallets which the SMO can release from its reserved inventory.

This main-structure model is generally used for single day and then the time period is two. However, the model is applied to multi-day on the confines of the SDP because it takes several days to introduce and withdraw new SFN of the SDP. And then, the time period becomes more than two. To be specific, the new SFN is inputted into the last SFN (SFN) of the model when the new SFN should be introduced before the SDP. And original SFN is inputted into the last SFN (SFN) of the model when the SFN should be restored to the original SFN after the SDP.

This model is also a network flow model and can be solved in a polynomial time.