MPLS 기반 IP망에서 열선성 기법을 이용한 성도 설정 해법

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Column Generation Approach to the Constraint Based Explicit Routing Problem in MPLS Based IP Networks

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Abstract

We consider the constraint based explicit routing problem in MPLS based IP Network. In this problem, we are given a set of traffic demands and a network with different link capacities. The problem is to assign the demand commodities to the paths in the network while minimizing the maximum link load ratio.

We formulate this problem as an integer programming problem and propose an efficient column generation technique. To strengthen the formulation, we consider some valid inequalities. We also incorporate the column generation technique with variable fixing scheme. Computational results show that the algorithm gives high quality solutions in a short execution time.

1. Introduction

The Internet of today is evolving rapidly. The importance of internet for the communications has been recognized by a great number of users, and Internet is considered as one of the drivers for our new global international society.

The growth in the number of users of the Internet and their bandwidth requirements makes traffic congestions on the Internet service providers' (ISPs) networks. To meet the growing demand for bandwidth and guaranteed Quality of Service (QoS), ISPs need higher performance switching and routing technology [1].

Multiprotocol label switching (MPLS) is one such technology. MPLS offers simple mechanisms for packet oriented traffic engineering and multisevice functionality with the added benefit of greater scalability. MPLS emerged from the Internet Engineering Task Force (IETF)'s effort to standardize a number of proprietary multilayer switching solutions that were initially proposed in the mid-1990s [11]. MPLS uses short, fixed length, locally significant labels in the packet header and packets are forwarded by network nodes via label swapping similar to layer 2 switching. MPLS is intended to work over any data link layer technology including ATM, frame relay, PPP and Ethernet. A router that supports the MPLS protocol is called a Label Switching Router (LSR). In MPLS, packet flows are mapped to Forwarding Equivalence Classes (FEC) which are mapped to traffic trunks, which in turn are mapped to Label Switched Paths (LSP) [3].

We now consider constraint based routing which is one of the most important innovations enabled by MPLS. The constraint based routing is a step beyond conventional IP routing because, in addition to minimizing some administrative metrics, it also selects paths that satisfy one or more constraints. Typical constraints include the available bandwidth along the path and administrative constraints.

Support for constraint based routing requires explicit routings (or source routing) capability. To provide this capability, we use MPLS based
explicit routing capability. The reason for using MPLS are twofold. First of all, MPLS allows
decoupling of the information used for forwarding (a label) from the information carried in
the IP header. Second, mapping between an FEC and an LSP is completely confined to the
Label Edge Router (LER), where LER is the
LSR at the head end of the LSP [1].

This explicit routing feature of MPLS may
overcome many shortcomings associated with
current Interior Gateway Protocol (IGP) routing
schemes. A prime problem of current IGP
routing schemes is that some links on the
shortest path between certain ingress egress pairs
may get congested while links on possible
alternative paths remain free.

Therefore routing schemes that can make
better use of network infrastructure are needed.
In the MPLS network, explicit routing with
Resource Reservation Protocol (RSVP)-extensions
allows the network to be able to control the
path from ingress node to egress node so as to
optimize utilization of network resources and
enhance performance [6].

In an operational MPLS network, the
routing computations implemented in edge
routers automate LSP setups. However, edge
routers have only local information about the
network resources. These local routing decisions
made in LERs may result in a degraded global
network performance in the long run. One
solution for this is to perform periodical (not
too frequent, e.g. daily or weekly) global
re-optimization of LSPs with a centralized
off-line network optimization tool [5].

In this study, we consider such a global
optimization algorithm to solve an explicit
routing problem in MPLS based IP network.
The problem we try to solve is to place
bandwidth guaranteed explicit routes between
e nodes over a physical topology such that
all the traffic demands are fulfilled. We also
consider the hop constraint for each explicit
route to ensure delay factor of QoS. The
optimization objective we propose is to minimize
the maximum of link load ratio. This
optimization objective ensures that the traffic
may be moved away from congested hot spots
to less utilized parts of the network, and the
distribution of traffic is balanced as much as
possible across the network. Minimizing the
maximum of link load ratio also leaves more
space for future traffic growth. Therefore, the
growth in traffic in the future are more likely to
be accommodated, and can be accepted without
requiring the re arrangement of connections [13].

To obtain these QoS routings and to solve
traffic engineering problems, there has been
several researches. For example, Wang and
Crowcroft [12] and Guerin and Orda [4] have
proposed a QoS routing algorithm, but in the
greedy style. Xiang et al. [14] have proposed
the scheme of QoS routing based on a genetic
algorithm.

Recently, Wang and Wang [13] have
proposed 4 heuristic algorithms for the explicit
routing in the MPLS based IP network. But
they have not considered any QoS constraints.
Also, Girish et al. [3] have formulated the
optimization problem for constraint based routing
to determine the optimal placement of a set of
LSPs in a network, but without any solution
approach.

In this study, we propose a column
generation approach to the explicit routing
problem for the traffic engineering in the MPLS
based IP network. Our approach is different
from the above researches in the sense that we
consider link utilization as optimization objective
and provide high quality solution using the
column generation approach.

The rest of this paper is organized as
follows. In section 2, we formulate the problem.
In section 3, we propose some valid inequalities
used in the preprocessing. In section 4, we
present the column generation procedure. Section
5 describes the overview of our algorithm.
Computational results are shown in section 6.
Finally, concluding remarks are given in section
7.

2. Formulation of the Problem

In this section, we present the mathematical
formulation of constraint based routing
problem (CRP). In this problem, we assume that
the backbone network is consists of a set of
nodes (LSRs) connected by directional links with
fixed capacities. Thereby, the network which we
consider is a directed graph. We also assume
that the average bandwidth demand from one
degree node to another is known. This demand is
measured by ISPs, or in the case of VPNs,
specified by customers for the logical
connection. Note that an explicit route must start
from one edge node (ingress LSR) and terminate
at another edge node (egress LSR) in MPLS
networks. Following are notations and decision
variables to be used for modeling.
Notations

\( N \) : the set of nodes in the network
\( E \) : the set of links in the network (which are defined as directed arcs)
\( K \) : the set of traffic demands between a pair of edge nodes.
\( d_k \) : effective or equivalent bandwidth of a traffic demand \( k \).
\( c_{ij} \) : bandwidth or available bandwidth of a link \((i,j) \in E\)
\( h_k \) : maximum allowed number of LSR hops through the network for a LSP \( k \in K \)
\( s_k \) : ingress LSR (source node) of a traffic demand \( k \in K \)
\( t_k \) : egress LSR (destination node) of a traffic demand \( k \in K \)
\( F(k) \) : the set of \((s_k, t_k)\) paths of traffic demand \( k \) such that the number of hops of path \( p \) is less than \( h_k \)
\( E_k \) : the set of links of which path \( p \) consists
\( a \) : decision variable that shows link load ratio

Then, the problem can be formulated as follows.

\[
\begin{align*}
\text{(MP)} & \quad \min \quad a \\
\text{s.t.} & \quad \sum_{p \in F(k)} d_k y_p^k \leq c_{ij}, \quad \forall (i,j) \in E \\
& \quad \sum_{k \in K} y_p^k = 1, \quad \text{for } k \in K \\
& \quad y_p^k \in \{0,1\}, \quad \text{for } p \in F(k), k \in K.
\end{align*}
\]

In this formulation, the variable \( y_p^k \) is 1 if and only if traffic demand \( k \) is assigned to path \( p \), otherwise 0. The objective (1) says the variable to be minimized is the maximum of link load ratio. Constraints (2) ensure that the link capacities are not exceeded. Constraints (3) imply that there must exist a path for each traffic demand.

Note that this CRF is NP-hard. It can be shown by reducing the \( k \) disjoint route problem which is NP hard [2] to this problem. For the details of this procedure, refer to Wang and Wang [13].

In our study, we use the efficient column generation technique to solve MP. The column generation is a pricing scheme for solving large scale linear programs. The column generation technique has usually been used to solve the problems whose natural formulations have exponential number of variables.

3. Preprocessing

In this section, we propose an efficient procedure which is used to strengthen the initial formulation. First, we consider the result of W. Yang [15], who has considered following proposition and incorporated it with his algorithm to increase the efficiency of the heuristic algorithm.

Proposition 1([15]). Following inequality is valid

\[
a \geq \max_{k \in K} d_k / \max_{(i,j) \in E} c_{ij}
\]

Now, we consider another valid inequality. First, we define a subset \( X_k \) of \( E \), which is defined as links that are adjacent to node \( k \in K \). By considering the capacities of links that are adjacent to the source node or destination node of traffic demand \( k \), we can obtain following results.

Proposition 2. Following inequality is valid

\[
a \geq \max_{k \in K} \left\{ \frac{d_k}{\max (\tilde{c}_{ik}, \tilde{c}_{jk})} \right\}
\]

where \( \tilde{c}_{ik} = \max_{(i,j) \in \Delta(k)} c_{ij} \), and \( \tilde{c}_{jk} = \max_{(i,j) \in \Delta(j)} c_{ij} \).

Proof. Note that traffic demand \( k \) originates from \( s_k \) and terminates at \( t_k \). Since the demand \( k \) paths through one link among \( \Delta(s(k)) \) and one link among \( \Delta(t(k)) \) \( a \) should satisfy that (i) \( a \geq d_k / \tilde{c}_{ik} \) and (ii) \( a \geq d_k / \tilde{c}_{jk} \). Therefore we have \( a \geq d_k / \min (\tilde{c}_{ik}, \tilde{c}_{jk}) \). Note that we are given a set of traffic demands. Therefore \( a \) should satisfy this inequality for each traffic demand \( k \in K \). Then we obtain the validity of the inequality (5). □

Proposition 3. The valid inequality (5) gives tighter bound than that of the valid inequality (4).

Proof. Let \( \tilde{k} = \arg \max_{k \in K} d_k \). For the \( \tilde{k} \) we define \( \tilde{c}_{ik} = \max_{(i,j) \in \Delta(k)} c_{ij} \) and \( \tilde{c}_{jk} = \max_{(i,j) \in \Delta(j)} c_{ij} \). Now we consider two cases.
(i) If \( \tilde{c}_{ik} \) and \( \tilde{c}_{jk} \) are not same, then

\[
\max_{(i,j) \in E} c_{ij} > \tilde{c}_{p}
\]

where \( \tilde{c}_{p} = \min (\tilde{c}_{ik}, \tilde{c}_{jk}) \).
\[ \hat{a}_2 \). Therefore, \( \max_{k \in K} d_k / \max (\hat{a}_k \in E) \leq d / \hat{c}_2 \) \( \leq \max_{k \in K} (d_k / \min (\hat{a}_k \in E)) \). (ii) If \( \hat{c}_7 \) and \( \hat{a}_2 \) are same, then \( \max (\hat{a}_k \in E) \geq \hat{c}_7 \). Therefore, we obtain \( \max_{k \in K} d_k / \max (\hat{a}_k \in E) \leq d / \hat{c}_2 \leq \max_{k \in K} d_k / \min (\hat{a}_k \in E) \). Therefore we complete the proof.

4. Column Generation Problem

In this section, we give an explanation of column generation problem and the algorithm to solve the problem. To represent the column generation procedure for the LP relaxation of MP, let MPL be the linear programming relaxation of MP. We need to have a feasible basis for MPL to use the column generation method. If it is difficult to find an initial feasible solution, we introduce artificial variables with big cost coefficient. We will mention how to find an initial feasible solution to MPL in section 5.

Given a feasible basis to MPL, we need to generate columns to enter the basis. We assume that a subset \( F(\hat{k}) \subseteq F(k) \) of path set for each \( k \in K \) is given. Replacing \( F(k) \) by \( F(\hat{k}) \) for all \( k \in K \) in MPL yields the restricted linear programming MPL', whose solutions are suboptimal to MPL. Let \( \beta_{ij} \) be the dual variable associated to the constraints (2) for each link \( (i,j) \in E \). Let \( \theta_k \) be the dual variable associated to the constraints (3). For the given \( k \in K \), the constraints in the dual of MPL' are

\[ \theta_k + \sum_{(i,j) \in E} d_k \beta_{ij} \geq 0, \quad (i,j) \in E \]

Let \((\beta, \theta)\) be an optimal solution to the dual of MPL'. Then it is also optimal to the dual of MPL if

\[ \theta_k + \sum_{(i,j) \in E} d_k \beta_{ij} \geq 0, \quad (i,j) \in E \]

Therefore, we may write the optimality condition for MPL:

\[ \min \{ \sum_{(i,j) \in E} (d_k \beta_{ij}) \mid (i,j) \in E \} = \theta_k \quad (6) \]

Using (6), we can derive the formulation of the column generation problem for MP. For the given \( k \in K \), the column generation problem associated to traffic demand \( k \) can be formulated as follows.

\[
\begin{align*}
\text{min} & \quad \sum_{(i,j) \in E} d_k \beta_{ij} \\
\text{s.t.} & \quad \sum_{\delta(i) \in \delta(j)} x_{ij} \leq h_k \\
& \quad \sum_{\delta(i) \in \delta(j)} x_{ij} = 0, \text{ for } i \neq s_k, k \\
& \quad \sum_{\delta(i) \in \delta(j)} x_{ij} = 1, \text{ for } i = s_k \\
& \quad x_{ij} \in \{0,1\}, \text{ for } (i,j) \in E 
\end{align*}
\]

(7)

In this formulation, the variable \( x_{ij} = 1 \) if and only if traffic demand \( k \) is routed on link \((i,j) \in E\). Otherwise, Constraints (7) imply that the number of LSR hops of the given traffic demand should be less than the bound \( h_k \). Constraints (8) and (9) imply that there must exist a path for the given traffic demand.

Note that SP(\( k \)) is the problem that finds the shortest path from \( i \) to \( j \), with hop constraint, where \( i \) is the source and \( j \) is the destination of traffic demand \( k \in K \), respectively. Since the link weights are nonnegative, SP(\( k \)) can be solved efficiently by Bellman-Ford's algorithm [7]. If the resulting length of the shortest path is less than \( -\theta_k \), the path can be added to the current formulation. Otherwise, no column is generated with respect to traffic demand \( k \).

5. The Algorithm

5.1 Overview

In this section, we give a brief and overall explanation of our algorithm. First, we construct the initial formulation of LP using artificial variables with big cost coefficient and initial feasible solutions.

After preprocessing and solving the initial LP, we decide whether the present solution is dual feasible or not. If it is not, new columns are generated and added to LP. We repeat this procedure until no more columns are added. If the present solution is dual feasible, i.e., no more column need to be added to LP, the final formulation of LP relaxation of MP is obtained.

Then, we check if the solution obtained by
solving the last LP is integral. If we have obtained an integral solution, we have obtained an optimal solution of MP. Otherwise, we have to initiate the variable fixing procedure to find an integral solution. The procedure incorporates the column generation within the variable fixing scheme. The procedure yields an integral solution of MP.

5.2 Initial Feasible Solution

To use the column generation procedure, we need to have an initial feasible solution to the LP relaxation of MP. We can obtain an initial feasible solution by finding paths between the ingress routers and the egress routers, respectively, for all traffic demands. Note that an initial feasible solution, if found, can also serve as an incumbent solution in the variable fixing procedure.

To find an initial feasible solution, we used following approach. We first try to find a path between the ingress router and the egress router for each traffic demand by using the Dijkstra's shortest path algorithm. If we can get a path that also satisfies the hop restriction, there is no problem. Otherwise, we apply the Bellman-Ford method [7] to find the path that satisfy the hop restriction. We repeat this procedure for all traffic demand $k \in K$.

Note that the computational complexity of Bellman-Ford method is $O(N^2 L)$, where $L$ is the number of maximum hops between source and destination pair. This algorithm is much more time consuming than Dijkstra's algorithm whose complexity is of $O(N \log N)$.

5.3 Primal Heuristic

In this section, we present simple heuristic algorithm to find an incumbent solution. The incumbent solution can be obtained by the fixing of decision variables during the variable fixing phase, or by the primal heuristic algorithm from the start of the variable fixing phase. In our algorithm, we developed greedy style heuristic algorithm as follows to be incorporated at the start of the variable fixing phase.

**Algorithm**: LHA (LP-based Heuristic Algorithm)

**Step 0**: Initialization Set $l_{ij} = 0$ for all $(i,j) \in E$.

**Step 1**: Solve LP. Solve LP relaxation of MP using the column generation algorithm. Let $y^*$ be the obtained optimal solution.

**Step 2**: Path assignment. For all $k \in K$, find $\tilde{P}_k$, where $y^*_{k,i} = \max\{y^*_{k,i} | d \in \tilde{P}_k, k \in K\}$. Set $l_{ij} = l_{ij} + d_{ij}$ for all $(i,j) \in E$.

**Step 3**: Maximum load ratio computation. Find maximum load ratio $\hat{I}_{ij}$, where $\hat{I}_{ij} = \max\{l_{ij} | (i,j) \in E\}$

Note that $\hat{I}_{ij}$ is an upper bound on the objective value of MP. The number of iterations needed by the algorithm is $|K|$. Now, in the next section, we present variable fixing phase as follows.

5.4 Variable Fixing Phase

When we solve the LP relaxation of MP by the column generation, we may get the fractional solution. In order to get an integral solution, we need to consider branching strategies. Basically branch and price procedure can be considered in order to obtain an optimal solution. But, it generates exponential number of branching nodes. Therefore we can not guarantee the termination of the algorithm. To overcome these drawbacks in this study, we use the following variable fixing strategy which is based on one directional branch scheme.

When we get a fractional optimal solution of the LP relaxation of MP, we first select a variable with maximum value among path variables $y^*$ that satisfy the hop restriction. After the variable selection, we fix that variable's value to one. It means that the lower bound of that variable is changed to one. After that, we solve the LP relaxation problem of MP and then again solve the subproblem to generated columns until no more columns are generated. After completing this procedure, if the LP relaxation solution of MP is integral, we are done with an integral feasible solution. Otherwise we repeat this procedure until we get an integral solution. In this way, we finally find an integral feasible solution.

6. Computational Results

In this section, we present the performance of
the proposed algorithm. We first mention the characteristics of the generated problems. Then, we give the computational results of our algorithm.

6.1 Problem Characteristics

We tested the proposed algorithm on some randomly generated problems. We first generate two networks. The underlying networks are randomly generated from a discrete uniform 50 × 50 Euclidian plane.

For the given network topology, we construct 8 test classes with the number of demands varying from 300 to 1000 in 100 increment. We generate 10 test problems for each class. For each test problem, the link capacities are generated randomly among [800, 900, 1000, 1100, 1200]. The size of each individual demand is generated by a random variable with a uniform integer distribution in [1, 10]. The source and destination pairs are selected also randomly among all edge nodes.

Table 1 summarizes the characteristics of the randomly generated data. The column #Dem shows the number of demands. The column "Tot Demand" is the average of the total bandwidth requirement of all demands in the network for 10 test problems. The column "Tot Capacity" is the average of the total capacity of all links in the network for 10 test problems.

<table>
<thead>
<tr>
<th>(N, E) No.</th>
<th>#Dem</th>
<th>Tot Demand</th>
<th>Tot Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>1643</td>
<td>93010</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>2119</td>
<td>92170</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>2738</td>
<td>92310</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>3386</td>
<td>91760</td>
</tr>
<tr>
<td>(36, 92)</td>
<td>5</td>
<td>3674</td>
<td>91320</td>
</tr>
<tr>
<td>6</td>
<td>800</td>
<td>4401</td>
<td>90360</td>
</tr>
<tr>
<td>7</td>
<td>900</td>
<td>4928</td>
<td>92470</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>5520</td>
<td>92940</td>
</tr>
<tr>
<td>1</td>
<td>300</td>
<td>1636</td>
<td>150770</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>2191</td>
<td>149920</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>2751</td>
<td>149780</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>3001</td>
<td>150640</td>
</tr>
<tr>
<td>(45, 150)</td>
<td>5</td>
<td>3675</td>
<td>150040</td>
</tr>
<tr>
<td>6</td>
<td>800</td>
<td>4427</td>
<td>150530</td>
</tr>
<tr>
<td>7</td>
<td>900</td>
<td>4914</td>
<td>149420</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>5481</td>
<td>148820</td>
</tr>
</tbody>
</table>

For an LP solver and constraints addition routine, we use the CPLEX 7.0 callable mixed integer library. All tests are performed on a Pentium PC (866MHz).

6.2 Computational Results

The computational results on the 2 networks are summarized in tables 2-3.

In those tables, the column #Dem shows the number of traffic demands. The column #HOP shows the maximum allowed number of TLR hops through the network for a traffic demand. Let MH be the minimum number of hops by which the demand can be transported. For each demand, the hop restrictions T1, T2, T3 are given by \( \lceil \frac{T}{3} \rceil \), 2MH, and 3MH respectively. The column #COL shows the number of the columns that include the columns generated and the initial columns. The column FLP shows the objective value obtained by solving the final linear programming relaxation of LP before the variable fixing procedure and the column IP denotes the value of the best integer solution. Then, Gap(%) is defined as follows:

\[
\text{Gap}(\%) = \frac{(\text{LP-FLP})}{\text{IP}} \times 100.
\]

Finally, the column Time refers to the execution time in second needed to solve the problem until variable fixing procedure. Each row shows the average value of 10 problem instances.

Usually, the number of columns generated is relatively large. Note that the number of columns generated increases as the number of demands grows. The Gap is relatively small and almost within 4 percent. It can also be observed that the Gap does not degrade as the number of demands grows.

Note that the maximum link load ratio increases when more demands are added to the network. As we have expected, the tables show that the result of the problems with loose hop restriction show better load balancing than that of the problems with tight hop restriction. But, the tables show that the Gap of the problems with loose hop restriction is not always smaller than the Gap of the problems with tight hop restriction.

The execution times needed to solve the problem of sized, \( |K| \) up to 1000, do not exceed 3 minutes. The average execution time is within 90 seconds.
7. Conclusions

In this paper, we proposed integer programming models for the constrained explicit routing problem in MPLS based IP networks. To solve this problem, we applied column generation approach and the variable fixing procedure. To strengthen the LP relaxation of the problem, we considered some valid inequalities in the preprocessing procedure. A primal heuristic algorithm is applied to find an incumbent solution.

Computational results show that the proposed algorithm can provide high quality solutions in short execution time. The proposed algorithm can be applied to solve several kinds of routing problems that consider not only hop constraints but also the other QoS attributes such as packet loss or jitter.

References


