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Comparison of wavelength requirements between two wavelength assignment methods in survivable WDM networks

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Abstract We consider the routing and wavelength assignment (RWA) in survivable WDM network. A path protection scheme assumed and two different wavelength assignment methods for protection paths are considered. Integer programming formulations of RWA under two wavelength assignment methods are proposed and we devised algorithms to solve them. Test results show that the difference of wavelength requirements between two wavelength assignment methods is 5–30%.

Keywords Integer programming · WDM · RWA · Column generation

Introduction

Wavelength division multiplexing (WDM) technology is used to accommodate several wavelength channels on a fiber. An all-optical network based on WDM is considered as a very promising approach for the realization of future large bandwidth networks (Lee, Lee, and Park, 2000). In a WDM network without wavelength conversion, an optical path (*lightpath*) with a dedicated wavelength is established for each required connection and no two paths using the same wavelength pass through the same link to avoid collision. The RWA problem is how to realize the required connection among nodes without wavelength collision. It is crucial for network planning and design to determine the wavelength requirements. Many studies on RWA have been performed (Chlamtac, Ganz, and Karmi, 1992; Kurma and Kurma, 2002; Lee et al., 2000).

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Survivability is an ability to recover the traffic when a network component fails and it has been an important issue in designing a fiber-optic based telecommunication network. In a layered transmission network such as WDM network, several layers (such as SONET, ATM and IP) may have their own recovery procedures. However, the recovery time for higher layers (such as ATM and IP) is still significantly large (on the order of seconds), whereas we expect that restoration times at the optical layer will be on the order of milliseconds to minimize data losses (Bonefant, 1998). Furthermore, it is beneficial to consider protection mechanisms in the optical layer for the following reasons: (a) the optical layer can efficiently multiplex protection resources among several higher-layer network applications, and (b) survivability at the optical layer provides protection to higher-layer protocols that may not have built-in protection (Ramaswami, 2000).

Optical layer protection schemes in WDM networks can be implemented at either path layer or link layer and there are several protection schemes. We refer to Gerstel and Ramaswami (2000) for detailed characteristics and comparisons of those schemes.

In wavelength assignment for protection paths, there are two possible methods (Mohan and Murthy, 2000; Nagatsu, Okamodo, and Sato, 1996). One method assigns an arbitrary wavelength for each protection path (method-DIFF). The other method assigns the same wavelength to the protection paths as its corresponding working path (method-SAME). In method-SAME, because of the more strict limitation on wavelength reuse throughout the network, the inefficient use of network resources may be caused. On the other hand, in method-DIFF, the higher the bit rate of lightpath, the more significantly the protection performance is affected by the processing capability of the electrical level switching (Nagatsu, Okamodo, and Sato, 1996).

In this paper, we show the difference of wavelength requirements between the two methods. To get the wavelength requirements, we solve the RWA under the two methods when the same working paths are given. We assume the single-link failure scenario and a path protection scheme in optical layer. When a failure occurs, the event is quickly disseminated to all the pertinent nodes, which set up the predetermined protection paths for the failed lightpaths and switch data to them (Nagatsu, Okamodo, and Sato, 1996). The protection path for each working path is independent to the location of the failure. In other words, a working path and the corresponding protection path are link-disjoint. We call the RWA problem under the two protection methods SRWA-I and SRWA-II, respectively.

Many studies on RWA considering survivability have been performed. Nagatsu, Okamoto, and Sato (1996) decompose the problem into two subproblems and solve them sequentially. One is to construct working paths and the other is to construct protection paths. They proposed a heuristic procedure for each subproblem. Miyao and Saito (1998) proposed an integer programming formulation considering a path protection when full wavelength conversion is permitted in every nodes. Then, the problem is free from wavelength assignment for each path. They assume that the set of possible pairs of a working path and corresponding protection path is given and solve the formulation by CPLEX. Ramamurthy and Mukherjee (1999) proposed an integer programming formulation which contains path variables. But, their formulation is huge and does not contain all constraints for wavelength assignment. They tried to solve the formulation with a small sized problem instance but they could not obtain an optimal solution. Modiano and Tam (2001) consider the problem to setup lightpaths for preserving 2-connected network after a link failure. They proposed an integer programming formulation and solve with CPLEX. But, their formulation does not consider the wavelength assignment for lightpaths. Reddy, Manimaran, and murthy (2000) consider the problem to reconfigure all the existing lightpaths not to reroute only the failed lightpath when a link failure is occurred. They propose a restoration scheme based on a heuristic rerouting procedure to minimize

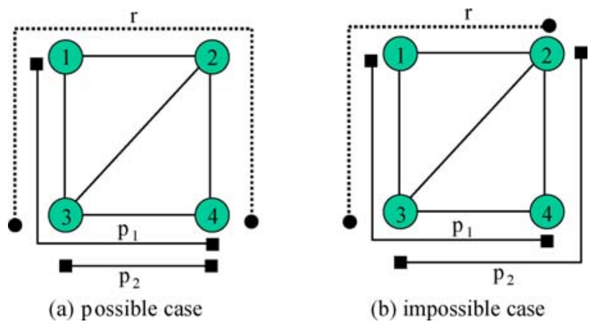

Fig. 1 Two cases of wavelength assignment on a working path and a protection path

blocked lightpaths. Narula-Tam, Lin, and Modiano (2002) considers the RWA on WDM ring networks. They calculated the lower bound of required wavelengths and proposed routing and wavelength assignment algorithms.

In this paper, we give integer programming formulations to SRWA-I and SRWA-II using the concept of routing configuration described in the following section. A similar concept is used in our other papers (Lee, Lee, and Park, 2000; Lee et al., 2000) but we did not consider the survivability in those papers. The routing configurations have distinct properties because of survivability and it is quite different to obtain the configuration. The formulations have exponentially many variables but we can solve the linear programming (LP) relaxation of them by column generation technique (Barnhart et al., 1998; Savelsbergh, 1997). The column generation problems are also NP-hard. But, we formulate them and we propose an branch-and-price algorithm to solve them. After solving the LP relaxation, we propose a variable fixing procedure combined with column generation to obtain an integral solution.

This paper is organized as follows. In Section 1, we describe the problem and we give integer programming formulations. We explain the algorithm in Section 2. In Section 3, we show computational results and conclusions are given in Section 4.

1. Mathematical model

In this section, we propose integer programming formulations of SRWA-I and SRWA-II. Consider an undirected mesh network $G = (V, E)$ where V is a given node set and E is a given edge set. When a set of working paths P is given, we select a link-disjoint protection path for each working path and assign a wavelength for each working and protection path. The objective is to minimize the number of required wavelengths for maximizing the wavelength reuse. Now, we introduce some notations.

o_p, d_p: two end nodes of working path $p \in P$

$R(p)$: set of all possible protection paths for working path $p \in P$

$R = \cup_{p \in P} R(p)$

$E(p)$: set of links used by path p

To avoid wavelength collision, no two paths passing the same link use the same wavelength. But, the constraint on protection paths can be relaxed as follows under method-DIFF.

- Two protection paths sharing some links can use the same wavelength if the two corresponding working paths don't share any link. Suppose that two working paths share no links. Then, the working paths don't be failed at the same time under the single-link failure and the protection paths for them need not be activated at the same time. Thus, the protection paths can use the same wavelength though they share some links.
- A working path and a protection path sharing some links can use the same wavelength if the working path is failed whenever the protection path is activated. Let's consider working path p_1 and protection path r for working path p_2 in Fig. 1. In (a) and (b), p_1 and r share link $(1, 3)$ and p_1 also shares a link with p_2. In (a), if p_2 is failed then p_1 is also failed. In other words, p_1 is also failed whenever r is activated. Thus, p_1 and r can use the same wavelength though they share link $(1, 3)$. But, in (b), if the link $(2, 4)$ is failed, then p_2 is failed and r is activated but p_1 is not failed. Thus, p_1 and r can't use the same wavelength.

To formulate SRWA-I, we introduce the concept of routing configuration which was used in Lee et al. (2000). Here, a routing configuration is a set of paths in the union of P and R. We call routing configuration c as *survivable independent routing configuration* (SIRC) if all the paths in c can be established using only one wavelength under the method-DIFF. In other words, the paths in an SIRC make no wavelength collision satisfying above relaxed condition. Then, SIRC c can be represented by a binary vector $(a_c, b_c) \in B^{2|P|}$. The pth element of a_c and b_c, denoted as a_{pc} and b_{pc}, are 1 if working path p and a protection path for p is contained in routing configuration c, respectively. Define C as the set of all SIRC's and we can formulate SRWA-I as the following integer program.

$$
\begin{aligned}
\text{(MP1)}\quad \min \quad & \sum_{c \in C} z_c \\
\text{s.t.} \quad & \sum_{c \in C} a_{pc} z_c \geq 1, \text{ for all } p \in P && (1)\\
& \sum_{c \in C} b_{pc} z_c \geq 1, \text{ for all } p \in P && (2)\\
& z_c \in \{0, 1\} \text{for all } c \in C
\end{aligned}
$$

Each decision variable $z_c = 1$, $c \in C$ if SIRC c is established, otherwise, $z_c = 0$. Constraints (1) and (2) ensure that all given working paths and a protection path for each working path should be established. Object function is the number of required wavelengths. MP1 does not decide the wavelength assignment of paths explicitly but we can easily obtain the wavelength assignment by assigning different wavelength to each SIRC c^* when $z_{c^*} = 1$.

We call a routing configuration *identical survivable independent routing configuration* (ISIRC) if all contained paths can be established using one wavelength under method-SAME. If working path p is contained in an ISIRC, then a corresponding protection path $r \in R(p)$ must be contained in the same routing configuration. Thus, if a link is used by a working path, then the link cannot be used by any other protection path because the protection path can be activated. Otherwise, the link can be used by several protection paths because at most one of the protection path passing the link in the configuration can be activated at a time. Consider the examples in Fig. 2. Routing configuration (a) is an ISIRC containing two closed trails. Link (2, 3) is used by working path p_1 and then the link could not be used any other path. But, the link (1, 2) and (1, 3) are used by two protection paths because the two protection paths are not activated at the same time. Note that no two protection paths are activated at

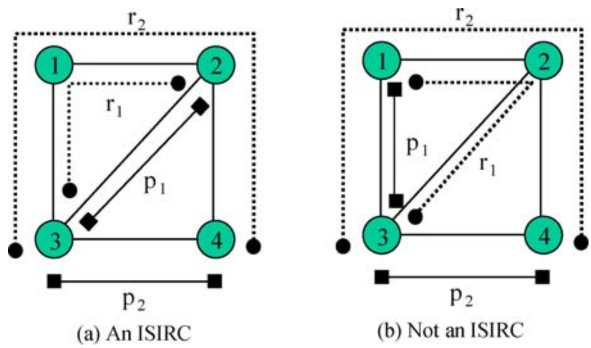

Fig. 2 Two routing configurations constructing new graph and paths

the same time in an ISIRC because the corresponding working paths must be in the ISIRC and they don't share any link. Thus, a link can be used by only one working path or arbitrary number of protection paths. So, the configuration (b) is not an ISIRC because link (1, 3) is used by working path p_1 and protection path r_2. When the link (3, 4) is failed, r_2 is activated and a collision occurs on link (1, 3).

If working path p is contained in an ISIRC, then a corresponding protection path r must be contained in the same routing configuration. A closed trail of a graph is a closed walk that traverses each link at most once. Because p and r are link-disjoint, those two paths form a closed trail. In other words, an ISIRC consists of several closed trails and each closed trail is divided into a working path and a corresponding protection path. Then, an ISIRC can be represented by a binary vector $d_c \in B^{|P|}$. The pth element of d_c, denoted as d_{pc} is 1 if a closed trail for p is contained in ISIRC c, otherwise, $d_{pc} = 0$. Define C' as the set of all ISIRC's and we can formulate SRWA-II as the following integer program.

$$
\begin{aligned}
\text{(MP2)} \quad \min \quad & \sum_{c \in C'} z_c \\
\text{s.t.} \quad & \sum_{c \in C'} d_{pc} z_c \geq 1, \quad \text{for all } p \in P \qquad (3) \\
& z_c \in \{0, 1\} \quad \text{for all } c \in C'
\end{aligned}
$$

Each decision variable $z_c = 1$, $c \in C'$ if ISIRC c is established, otherwise, $z_c = 0$. Constraints (3) ensure that at least one closed trail for each working path must be selected. It also means that all given working paths and protection paths corresponding them should be established. Like MP1, we can easily obtain the wavelength assignment by assigning the same wavelength to paths contained in an ISIRC c^* when $z_{c^*} = 1$. Then, each working and corresponding protection path use the same wavelength.

2. Algorithm

LP (Linear Programming) relaxation is obtained by dropping integral restriction on decision variables in an integer programming. Note that LP relaxations of MP1 and MP2 have exponentially many variables. Thus, it is impractical to solve them with all variables beforehand. However, we devised an algorithm to solve them efficiently by using column generation technique (Barnhart et al., 1998; Savelsbergh, 1997). Column generation technique starts with a restricted LP which contains a subset of variables at initial stage and needed variables are added later. Then, the key problem in the technique is to check whether the current solution is optimal or not and to find a needed variable when the solution is not optimal. The problem is called pricing problem.

2.1. Pricing problems

The pricing problems for MP1 and MP2 are to find a maximum weighted SIRC and ISIRC, respectively. They can be formulated following integer programming SP1 and SP2, respectively.

$$
\begin{aligned}
\text{(SP1)}\quad \max\ & \sum_{p\in P}\alpha_p^* x_p + \sum_{p\in P}\sum_{r\in R(p)} \beta_p^* y_r \\
\text{s.t.}\ & \sum_{r\in R(p)} y_r \le 1, \quad \text{for all } p\in P && (4)\\
& \sum_{p\in P} d^p_{e_1,e_2} x_p + \sum_{p\in P}\sum_{r\in R(p)} d^r_{e_1,e_2} y_r \le 1, \quad \text{for all } e_1, e_2 \in E && (5)\\
& x_p, y_r \in \{0,1\} \quad \text{for all } p\in P \text{ and } r\in R
\end{aligned}
$$

α_p^* and β_p^* are the values of the pth dual variables returned by the simplex method of (1) and (2) in restricted LP relaxation of MP1. The coefficient $d^p_{e_1,e_2} = 1$ if $e_1 = e_2$ or working path p passes link e_1 but it does not pass e_2, otherwise $d^p_{e_1,e_2} = 0$. Similarly, $d^r_{e_1,e_2} = 1$ if protection path $r \in R(p)$ passes link e_1 and corresponding working path p passes link e_2, otherwise $d^r_{e_1,e_2} = 0$. Each decision variable $x_p = 1$, $p \in P$ if working path p is selected, otherwise, $x_p = 0$. Decision variable $y_r = 1, r \in R$ if protection path r is selected, otherwise, $y_r = 0$. Constraints (4) ensure that at most one protection path for each working path can be established. Constraints (5) ensure that a feasible solution to SP1 is an SIRC. Suppose that two working paths p_1 and p_2 pass link e_1 then the coefficients $d^{p_1}_{e_1,e_1}$ and $d^{p_2}_{e_1,e_1}$ are 1 and both paths cannot be selected simultaneously. If two protection paths share link e_1 such that the corresponding working paths share link e_2, then $d^r_{e_1,e_2}$ for two protection paths are 1 and both paths cannot be selected simultaneously. Similarly, (5) satisfy the constraint between a working path and a protection path described in Section 2. Suppose that working path p_1 and protection path r for working path p_2 share link e_1. If there exists link e_2 such that p_2 passes e_2 and p_1 does not pass e_2, then p_2 is failed and p_1 is not failed when e_2 is failed. Thus, p_1 and r cannot use the same wavelength because r can be activated without failing of p_1. In that case, $d^{p_1}_{e_1,e_2}$ and $d^r_{e_1,e_2}$ are 1 in (5) and both paths cannot be selected simultaneously. As a result, a feasible solution to SP1 is an SIRC.

$$
\begin{aligned}
\text{(SP2)}\quad \max\ & \sum_{p\in P}\sum_{h\in H(p)} \gamma_p^* x_h \\
\text{s.t.}\ & \sum_{p\in P}\sum_{h\in H(p)} d^r_{eh} x_h - |P|(1-y_e) \le 0, \quad \text{for all } e\in E && (6)\\
& \sum_{p\in P}\sum_{h\in H(p)} d^p_{eh} x_h - y_e \le 0, \quad \text{for all } e\in E && (7)\\
& x_h, y_e \in \{0,1\} \quad \text{for all } h\in H \text{ and } e\in E
\end{aligned}
$$

γ_p^* is the value of the pth dual variable returned by the simplex method of (3) in restricted LP relaxation of (3). A closed trail for a working path p consists of working path p and a corresponding protection path $r \in R(p)$. The coefficient $d_{eh}^r = 1$ if the protection path r in closed trail h pass edge e, otherwise $d_{eh}^r = 0$. Similarly, $d_{eh}^p = 1$ if working path p in closed trail h pass edge e, otherwise $d_{eh}^p = 0$. $H(p)$ is the set of all possible closed trails on G which contain working path $p \in P$ and $H = \cup_{p \in P} H(p)$. Each decision variable $x_h = 1, h \in H$ if closed trail h is selected, otherwise, $x_h = 0$. Decision variable $y_e = 1, e \in E$ if edge e can be used by a working path, otherwise, $y_e = 0$. Constraints (6) ensure that link e may be used by multiple protection paths if the link is used by no working path. Constraints (7) ensure that at most one working path can pass on a link. (6) and (7) satisfy the restriction for an ISIRC described in the previous section, so a feasible solution to SP2 is an ISIRC.

As shown above, SP1(SP2) is the problem to find a maximum weighted SIRC(ISIRC). Now, we consider the computational complexity of SP1 and SP2. First, we can easily find that to get a set of working paths, such that no two paths share a link, which has maximum weight is a special case of SP1 and SP2. Consider the following decision problem DP associated with the special case.

Instance: Graph $G = (V, E)$, a given path set P, path weight w_p, $p \in P$, positive integer L
Question: Is there a subset of $P' \subseteq P$ such that no two paths in P' share a link and the sum of weights greater than or equal to L?

Now, consider the following INDEPENDENT SET problem which is NP-complete (Garey and Johnson, 1979).

Instance: Graph $G = (V, E)$, positive integer K.
Question: Does G contain an independent set of size K or more, i.e., a subset $V' \subseteq V$ such that $|V'| \geq K$ and no two vertices in V' are joined by an edge in E?

Proposition 1. *SP1 and SP2 are NP-hard.*

Proof: Given an arbitrary instance of the INDEPENDENT SET problem where $G = (V, E)$, we can construct a new graph $G' = (V_A, V_B, E')$ as follows.

1. Make center node v_0.
2. Let edge $i \in E$ be represented by two nodes $v_i \in V_A$ and $v_i' \in V_B$ and join them with a link. (Number the links in E from 1 to $|E|$)
3. Join the center node and all nodes in V_A and V_B with links.
4. Make a cycle with the nodes in V_A by adding links $(1, 2), (2, 3), \ldots, (|E| - 1, |E|), (|E|, 1)$.

Now, we present node $v \in V$ as path p_v on G' to have following properties. Denote $\delta(v)$ as the set of links in G which are incident to v.

1. p_v starts at v_0.
2. p_v passes link (v_i, v_{i+1}) ($(v_{|E|}, v_0)$ when $i = |E|$) in ascending order of i if and only if $i \in \delta(v)$.
3. p_v passes the link (v_0, v_i) if and only if it passes the link (v_i, v_{i+1}) ($(v_{|E|}, v_0)$ when $i = |E|$).

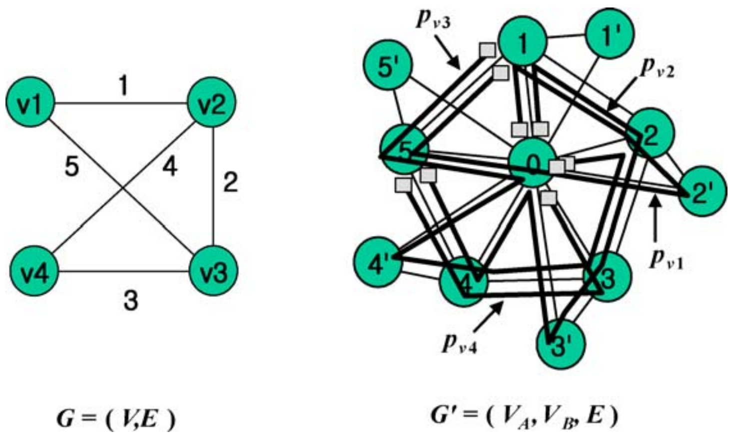

Fig. 3 Constructing new graph G' and paths

Then, if two nodes in G are joined by a link $i \in E$, then two paths on G' corresponding the two nodes share at least link (v_i, v_{i+1}) and there is no shared link between two paths on G' if and only if the corresponding nodes in G are not joined. Thus, it can be easily shown that there is one-to-one correspondence between solutions of independent set problem on G and solutions of DP on G' where $P = \{p_v\}$ for all $v \in V$, $L = K$ and $w_p = 1$ for all $p \in P$. An example of the transformation is shown in Fig. 3. It is clear that the transformation is performed in polynomial steps. Thus, DP is NP-compete and consequently SP1 and SP2 are NP-hard. $\square$

But, we devised following branch-and-price algorithms to solve them.

2.1.1. Column generation procedure for SP1

Now, we explain the column generation procedure to solve the LP relaxation of SP1. Let SLP1 be the LP relaxation of SP1 and let SLP1$'$ be a restricted LP relaxation obtained by replacing $R(p)$ in SLP1 by $R'(p) \subset R(p)$, for all $p \in P$. We construct initial SLP1$'$ with P and $R'(p) = \varnothing$ for all $p \in P$. Let ρ_p, for all $p \in P$, and δ_{e_1,e_2}, for all e_1 and $e_2 \in E$, be the nonnegative dual variables associated with constraints (4) and (5), respectively. We can solve SLP1$'$ by the simplex method and let (x^*, y^*) be the obtained optimal solution to SLP1$'$ and let $(\bar{\rho}, \bar{\delta})$ be the corresponding optimal dual solution. Then, the reduced cost of each protection path $r \in R(p)$, denoted c_p^r, is as follows.

$$c_p^r = \beta_p^* - \bar{\rho}_p - \sum_{e_1 \in E(r)} \sum_{e_2 \in E(p)} \bar{\delta}_{e_1,e_2}$$

If there exists no protection path whose reduced cost is positive, then current solution (x^*, y^*) is optimal. Thus, column generation problem is to find a protection path whose reduced cost is maximum. For a working path p, note that β_p^* and $\bar{\rho}_p$ are determined values and protection path $r \in R(p)$ must be link-disjoint with respect to p. Thus, the column generation problem for $p \in P$ is to find a shortest path between o_p and d_p on a given network $G = (V, E \setminus E(p))$ with the link weight $\sum_{e_2 \in E(p)} \bar{\delta}_{e_1,e_2}$ for $e_1 \in E \setminus E(p)$. Because the link weights are nonnegative, we can solve the problem in polynomial time. Denote the obtained shortest path as r_p^* and the length of r_p^* as l_{pr^*}. If $\beta_p^* - \bar{\rho}_p - l_{pr^*} \leq 0$ for all $p \in P$, then $(\bar{\rho}, \bar{\delta})$ is an optimal dual solution to the dual of SLP1 and (x^*, y^*) is an optimal solution to SLP1. Otherwise, we add r_p^* to SLP1$'$ for each $p \in P$ with $\beta_p^* - \bar{\rho}_p - l_{pr^*} > 0$ and solve SLP1$'$ again. We can solve SLP1 by repeating the above column generation procedure until no more column is generated.

2.1.2. Branch-and-price procedure for SP1

After solving SLP1, if the obtained optimal solution to SLP1 is integral then we solved SP1. Otherwise, we perform branch-and-price procedure to get an optimal solution to SP1. Branch-and-price procedure is the same as the branch-and-bound procedure except that the column generation procedure is used to solve SLP1 at every node in the branch-and-bound tree. In the branch-and-price procedure, a branching rule is required such that the column generation is possible after branching. Devising such a branching rule is an important key in the branch-and-price procedure. Refer Barnhart et al. (1998) for more details about the branch-and-price procedure.

Now, we present our branching rule which does not destroy the structure of the above column generation procedure after branching. When an optimal solution (x^*, y^*) to SLP1 is obtained, first we check whether $x_p^* \in \{0, 1\}$ for all $p \in P$. Denote $U(x^*, y^*) \subseteq P$ as the set of working paths such that corresponding x^* has fractional value. If $|U(x^*, y^*)| > 0$, we branch on x_{p^*} such that $p^* = \arg\max_{p \in U(x^*, y^*)} x_p^*$. We make two new branches, one of them would force x_{p^*} to be 1, the other would force x_{p^*} to be 0. In the column generation, no working path is generated. Thus, the column generation problem is not changed in those branches.

If $|U(x^*, y^*)| = 0$, then either (x^*, y^*) is integral or y^* is not integral. Suppose that y^* is not integral. We define $K(x^*, y^*; e_1, e_2) \subset P$ for all $e_1, e_2 \in E$ such that working path $p \in K(x^*, y^*; e_1, e_2)$ if and only if there exists a protection path $r \in R(p)$ with $y_r^* > 0$ such that $e_1 \in E(r)$ and $e_2 \in E(p)$. Then, the following proposition can be derived.

Proposition 2. *Suppose* (x^*, y^*) *is an optimal solution to SLP1$'$. When* $|U(x^*, y^*)| = 0$ *and* $|K(x^*, y^*; e_1, e_2)| \leq 1$ *for all* $e_1, e_2 \in E$ *then* $\sum_{r \in R(p)} y_r^* = 0$ *or* 1 *for all* $p \in P$.

Proof: Suppose that there exists $p^* \in P$ such that $0 < \sum_{r \in R(p^*)} y_r^* < 1$. Then there exists protection path $r^* \in R(p^*)$ such that $y_{r^*}^* > 0$. Let's consider the solution $(\bar{x}, \bar{y})$ such that $\bar{x}_p = x_p^*$ for all $p \in P$, $\bar{y}_r = y_r^*$ for all $r \neq r^*$ and $\bar{y}_{r^*} = 1 - \sum_{r \in R(p) \setminus r^*} y_r^*$. We can easily know that $(\bar{x}, \bar{y})$ is a feasible solution to SLP1$'$ because $|K(x^*, y^*; e_1, e_2)| \leq 1$ for all $e_1, e_2 \in E$. The objective value of $(\bar{x}, \bar{y})$ is greater than or equal to that of (x^*, y^*) because the objective coefficient $\beta_{p^*}^*$ in SLP1$'$ for protection path for p^* is greater than or equal to 0. If the objective coefficient is greater than 0, then it is a contradiction since (x^*, y^*) cannot be an optimal solution to SLP1$'$. If the objective coefficient is equal to 0, then no protection path for p^* has positive reduced cost. Then, no restoration path in $R(p^*)$ can be generated in column generation procedure and SLP1$'$ contains no path in $R(p^*)$. Thus, it is also a contradiction because $\sum_{r \in R(p^*)} y_r^* = 0$. □

Clearly, if (x^*, y^*) is an integral solution, then $|U(x^*, y^*)| = 0$ and $|K(x^*, y^*; e_1, e_2)| \leq 1$. Converse is not always true. However, when we obtain (x^*, y^*) by the simplex method we can derive the following positive result from Proposition 2.

Proposition 3. *Suppose an optimal solution* (x^*, y^*) *to SLP1′ is obtained by the simplex method. When* $|U(x^*, y^*)| = 0$, *if* $|K(x^*, y^*; e_1, e_2)| \leq 1$ *for all* $e_1, e_2 \in E$ *then* (x^*, y^*) *is integral.*

Proof: Clearly, x^* is integral. Suppose that y^* is not integral, then there exists p^* such that y_r^* for $r \in R(p^*)$ are not integral and $\sum_{r \in R(p^*)} y_r^* = 1$ by Proposition 1. Define $R(p^*; y^*) \subseteq R(p^*)$ such that $r \in R(p^*; y^*)$ if and only if $y_r^* > 0$. Consider the solutions $(\bar{x}, \bar{y})^s, s \in R(p^*; y^*)$ which are obtained as follows. We let $\bar{x} = x^*$, $\bar{y}_r = y_r^*$ if $r \notin R(p^*; y^*)$, $\bar{y}_r = 0$ if $r \in R(p^*; y^*) \setminus \{s\}$ and $\bar{y}_r = 1$ if $r = s$. It is easily verified that $(\bar{x}, \bar{y})^s, s \in R(p^*; y^*)$ are feasible solutions to SLP1′ because $|K(x^*, y^*; e_1, e_2)| \leq 1$ for all $e_1, e_2 \in E$. Moreover, $(x^*, y^*) = \sum_{s \in R(p^*; y^*)} y_s^* \cdot (\bar{x}, \bar{y})^s$ and $\sum_{r \in R(p^*)} y_r^* = \sum_{s \in R(p^*; y^*)} y_s^* = 1$. In other words, (x^*, y^*) is represented by a convex combination of other feasible solutions to SLP1′, which is a contradiction since the simplex method always finds an extreme point optimal solution. □

When x is integral ($|U(x^*, y^*)| = 0$), we only check if $|K(x^*, y^*; e_1, e_2)| \leq 1$ for all $e_1, e_2 \in E$ instead of checking the integrality of (x^*, y^*) by Proposition 3. For a given solution (x^*, y^*) such that $|U(x^*, y^*)| = 0$, we get $K(x^*, y^*; e_1, e_2)$ for all $e_1, e_2 \in E$. Then, we choose e_1^* and $e_2^* \in E$ with the lowest index such that $(e_1^*, e_2^*) = \arg\max_{e_1, e_2 \in E} |K(x^*, y^*; e_1^*, e_2^*)|$. If $|K(x^*, y^*; e_1^*, e_2^*)| = 1$, we get an integral solution by Proposition 3. Otherwise, we divide $K(x^*, y^*; e_1^*, e_2^*)$ into two disjoint sets $K_1(x^*, y^*; e_1^*, e_2^*)$ and $K_2(x^*, y^*; e_1^*, e_2^*)$, such that $K_1(x^*, y^*; e_1^*, e_2^*) = \{p^*\}$ and $K_2(x^*, y^*; e_1^*, e_2^*) = K(x^*, y^*; e_1^*, e_2^*) \setminus K_1(x^*, y^*; e_1^*, e_2^*)$, where $p^* = \arg\max_{p \in K(x^*, y^*; e_1^*, e_2^*)} \sum_{r \in R(p), e_1^* \in E(r)} y_r^*$ with the lowest index. Then we make two branches in the branch-and-bound tree such that any protection path for $p \in K_1(x^*, y^*; e_1^*, e_2^*)$ can not use link e_1^* in the first node and any protection path for $p \in K_2(x^*, y^*; e_1^*, e_2^*)$ can not use link e_1^* in the second node. That is, for the first node, we require

$$y_r = 0, \text{ for all } r \in R(p) \text{ such that } e_1^* \in E(r) \text{ and } p \in K_1(x^*, y^*; e_1^*, e_2^*)$$

and for the second node, we require

$$y_r = 0, \text{ for all } r \in R(p) \text{ such that } e_1^* \in E(r) \text{ and } p \in K_2(x^*, y^*; e_1^*, e_2^*).$$

To satisfy the above requirements, for each variable y_r which is already generated, we set the upper bound of the variable to 0 if $e_1^* \in E(r)$ for $r \in R(p)$ with $p \in K_1(x^*, y^*; e_1^*, e_2^*)$ in the first node, i.e. we set $y_r = 0$. We perform similar bound setting in the second node. Further, for each $p \in K_1(x^*, y^*; e_1^*, e_2^*)$, we perform the column generation procedure over the network obtained from removing e_1^* from the network $G = (V, E \setminus E(p))$ in the first node. Thus, we can generate columns by solving the shortest path problem after branching. For the second node, we apply the same scheme.

We can use the above branching rule by giving priority to x in branching. We branch on x first if there exist any one which has fractional value and we branch on y if x is integral.

2.1.3. Algorithm for SP2

We explained the branch-and-price algorithm for SP1. SP2 can be solved similar branch-and-price procedure. In solving SP2, column generation problem is to find a closed trail whose reduced cost is maximum. For a working path p, we must find a closed trail which contains p and we only find a protection path between o_p and d_p. Then, the column generation problem for $p \in P$ is to find a shortest path between o_p and d_p on a given network $G = (V, E \setminus E(p))$. Similarly SP1, we devised a branching rule which does not change the column generation problem structure and we can solve SP2 optimally. In the rule, we define $U'(x^*, y^*) \subseteq E$ as the set of links such that corresponding y^* has fractional value and $P(x^*, y^*; e) \subset P$ for all $e \in E$ such that working path $p \in P(x^*, y^*; e)$ if and only if there exists a closed trail $h \in H(p)$ with $x^*_h > 0$ such that $e \in E(p)$.

2.2. Overall algorithm with variable fixing

After solving LP relaxation of MP1 or MP2, if the obtained optimal solution is not integral, we can consider the branch-and-price procedure to obtain an optimal solution. But, it requires a method to prevent the regeneration of existing columns like the branching rules in above subsection. However, we can't devise such a branching rule and we consider only the case that a fractional variable is fixed to 1 and we generate more columns after fixing the variable. First, we select a variable which has maximum value among the variables having fractional

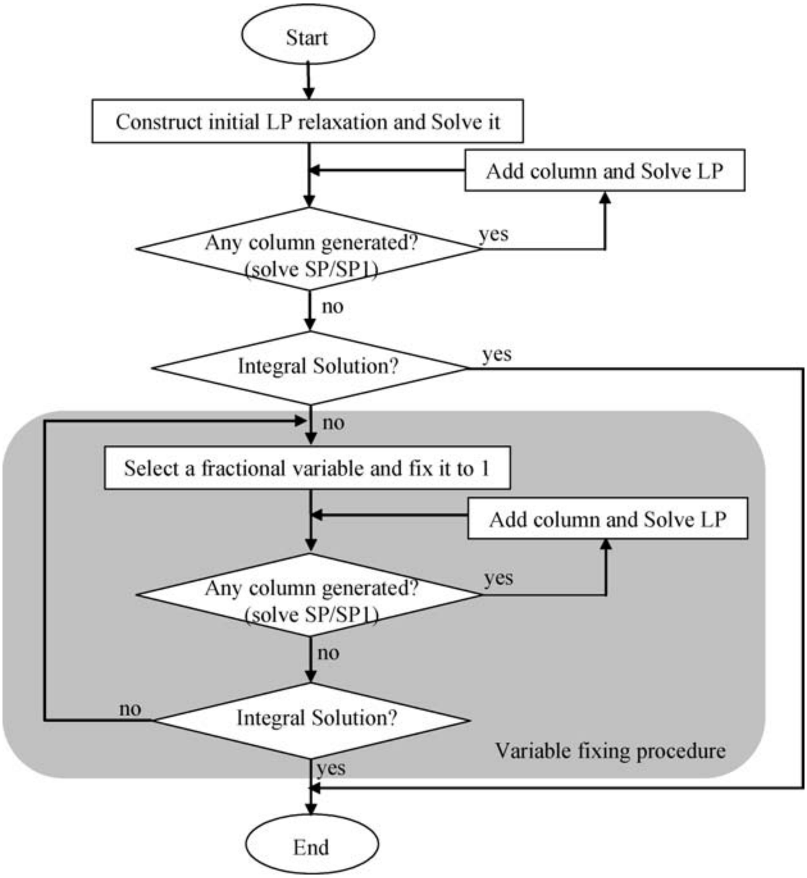

Fig. 4 Overall procedure of the algorithm

value in the last LP relaxation and then fixed the value of the variable to 1. After fixing, we solve the LP relaxation and we perform the column generation procedure until no more column is generated. If the obtained solution is integral then we have found an integral solution. Otherwise, we select another variable which has fractional value and fixed it to 1 and then generate columns again. We repeat above steps until we get an integral solution.

The procedure does not guarantee to find an optimal solution. But the final LP relaxation may contain many columns that are part of the optimal solution because the most profitable columns are generated and we generate more columns in the variable fixing procedure. Thus, we can expect to find a good solution. We can check the quality of our solution by comparing it with the lower bound obtained from the optimal value of LP relaxation. Computational results in the next section show that our solution is very good.

The flow chart of overall algorithm is given in Fig. 4.

3. Computational experiments

We tested our algorithm on two networks that are shown Figs. 5 and6. One is a 11-node network which is well cited in survivability studies and the other is the NSFNET.

For the test, we randomly generated working paths for all pairs of two nodes in each network. The number of working paths for each pair of nodes is 1 or 2 with the same probability. We use the shortest path between two nodes as the working path. We tested 20 randomly generated problem instances for each network. The tests were run on a pentium PC(700 MHz) and we used CPLEX callable mixed integer library as a LP solver. Test results are summarized in Tables 1 and2. In the table, the heading $\lceil Z_{LP-DIFF} \rceil$ and $\lceil Z_{LP-SAME} \rceil$

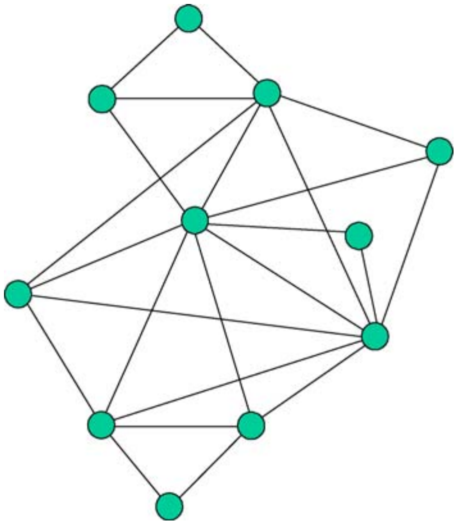

Fig. 5 The topology of 11-node test network

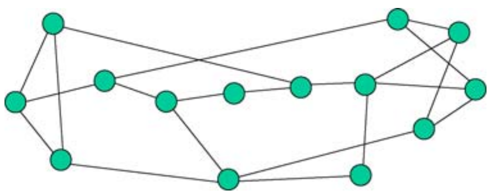

Fig. 6 National science fundamental network

Table 1 Computational results on 11-node network

No.	$\lceil Z_{LP-DIFF} \rceil$	Z_{M1}	Gap1	Time1 (sec.)	$\lceil Z_{LP-SAME} \rceil$	Z_{M2}	Gap2	Time2 (sec.)	Dif.	Dif (%)
1	16	16	0	51.79	18	18	0	65.20	2	12.5
2	16	16	0	48.55	17	17	0	51.68	1	6.3
3	17	17	0	28.18	20	20	0	33.57	3	17.6
4	15	15	0	14.89	16	16	0	33.93	1	6.7
5	19	19	0	43.94	20	20	0	117.53	1	5.3
6	18	18	0	13.23	19	19	0	51.03	1	5.6
7	15	15	0	24.99	16	16	0	103.45	1	6.7
8	18	18	0	32.08	19	19	0	266.43	1	5.6
9	16	16	0	11.37	18	18	0	81.57	2	12.5
10	19	19	0	27.30	20	20	0	38.70	1	5.3
11	19	19	0	10.60	21	21	0	57.62	2	10.5
12	19	19	0	11.10	20	20	0	59.93	1	5.3
13	18	18	0	10.57	19	19	0	147.61	1	5.6
14	18	18	0	16.09	19	19	0	126.81	1	5.6
15	18	18	0	28.86	19	19	0	70.32	1	5.6
16	19	19	0	22.90	20	20	0	75.62	1	5.3
17	17	17	0	31.08	20	20	0	134.40	3	17.6
18	19	19	0	14.07	20	20	0	66.64	1	5.3
19	16	16	0	17.52	17	17	0	160.39	1	6.3
20	16	16	0	30.09	18	18	0	48.48	2	12.5

Table 2 Computational results on NSFN

No.	$\lceil Z_{LP-DIFF} \rceil$	Z_{M1}	Gap1	Time1 (sec.)	$\lceil Z_{LP-SAME} \rceil$	Z_{M2}	Gap2	Time 2 (sec.)	Dif.	Dif (%)
1	25	25	0	381.24	28	28	0	323.02	3	12.0
2	20	20	0	492.91	25	25	0	2602.32	5	25.0
3	21	21	0	709.64	27	27	0	1328.04	6	28.8
4	20	20	0	977.56	23	23	0	2131.21	3	15.0
5	23	23	0	450.39	29	29	0	1616.34	6	26.1
6	22	22	0	639.78	25	25	0	999.15	3	13.6
7	24	24	0	749.19	29	29	0	1489.64	5	20.8
8	22	22	0	870.24	25	25	0	773.52	3	13.6
9	21	21	0	535.25	26	26	0	941.47	5	23.8
10	17	17	0	715.57	21	21	0	1731.36	4	23.5
11	20	20	0	562.22	25	25	0	725.72	5	25.0
12	25	25	0	588.75	29	29	0	3061.27	4	16.0
13	19	19	0	604.51	24	24	0	1485.41	5	26.3
14	20	20	0	963.56	24	24	0	1790.79	4	20.0
15	21	21	0	662.95	25	25	0	1435.70	4	17.3
16	22	22	0	696.57	28	28	0	1337.87	6	27.3
17	27	27	0	684.04	29	29	0	241.73	2	7.4
18	19	19	0	785.48	24	24	0	858.04	5	26.3
19	19	19	0	342.96	23	23	0	914.18	4	21.1
20	20	20	0	605.44	26	26	0	896.66	6	30.0

refer to the value obtained by rounding up the optimal objective value of LP relaxation of MP1 and MP2, respectively and they provides a lower bound on the optimal objective value. Z_{M1} and Z_{M2} refer to the objective value of SRWA-I and SRWA-II obtained by our algorithm. Gap1 and Gap2 are defined as Gap1 $= Z_{M1} - \lceil Z_{LP1} \rceil$ and Gap2 $= Z_{M2} - \lceil Z_{LP2} \rceil$ which give the upper bound on the difference optimal solution and solution obtained by our algorithm. The time to solve SRWA-I and SRWA-II by our algorithm are reported under the heading of Time1 and Time2, respectively. Dif is the difference of the wavelength requirements between method-DIFF and method-SAME.

As shown in the table, we get optimal solutions to all the test problems by our algorithm and the LP relaxations gives very tight bounds. Obviously, method-SAME requires more wavelengths. Our test results show that method-SAME requires about 6–30% more wavelengths than method-DIFF. On 11-node network, the difference is 6–18% and the difference is larger on NSFN. The 11-node network is more dense than NSFN and more paths exist between two nodes and it may cause the small difference. More experiments on various networks is required to know about the relation between the difference and network density.

4. Conclusions

We consider the routing and wavelength assignment problem on survivable WDM network. We proposed integer programming formulations of SRWA under two wavelength assignment methods and we devised algorithms based on column generation to solve them. We used the column generation technique to solve the LP relaxations of the problems efficiently. To solve the column generation problems, we developed branch-and-price algorithms. After solving the LP relaxation, we applied a variable fixing procedure combined with column generation to obtain an integral solution. We tested our algorithm on randomly generated data and our algorithm gives optimal solutions to all of the test problem instances.

In this paper, we considered the path protection schemes. Considering other protection schemes could be good research works. We assumed that the working paths are given, but the problem which decide the routing of working and protection paths together may be worth consideration. The problem may be very hard. However, any approach for the problem might be a worthwhile effort.

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