

## OR-1 Homework-4

Due : 2018/4/3 (Tue.), in class

1. A company manufactures two different products, 1, 2, using limestone as the basic raw material. The company has its own limestone quarries, which can produce up to 250 units of limestone per day at a cost of \$2/unit. If the company needs additional limestone, it can buy it from a supplier at a cost of \$5/unit.

The regional electric utility has recently adopted a modern stepwise rate system to discourage wastage. It charges the company \$30 per unit for the first 1000 units of electricity used daily, \$45 per unit for 500 units per day beyond the initial 1000 units, and a hefty \$75 per unit for any amount beyond the initial 1500 units of electricity used per day.

The region's water distribution authority charges at the rate of \$6 per unit of water used per day up to 800 units, and \$7 per unit for any amount used beyond 800 units per day.

The remaining data in the problem, including the selling price of the products, are tabulated below.

Formulate a linear programming problem to determine how much of each product to produce daily, so as to maximize the company's daily net profit. Explain the meaning of the variables you used.

Product	Units of Inputs Needed to Produce One Unit of Product			Selling Price of Product
	Limestone	Electricity	Water	
1	1/2	3	1	\$300/unit for the first 50 units; \$250/unit beyond 50 units per day
2	1	2	1/4	\$350/unit to an upper limit of 100 units/day

2. (Facility location problem)

Suppose we are given  $n$  potential facility locations and a list of  $m$  clients who need to be serviced from these locations. There is a fixed cost  $c_j$  of opening a facility at location  $j$ , while there is a cost  $d_{ij}$  of serving client  $i$  from facility  $j$ . The goal is to select a set of facility locations and assign each client to one facility, while

minimizing the total cost. Formulate the facility location problem as an integer program.

3. (A dynamic single-item lot sizing problem)

We consider the production of a single product over  $T$  periods. If we decide to produce at period  $t$ , a setup cost  $c_t$  is incurred. For  $t = 1, \dots, T$ , let  $d_t$  be the demand for this product in period  $t$ , and let  $p_t, h_t$  be the unit production cost and unit storage cost (per period), respectively. Assume that the initial inventory is 0.

Formulate an integer programming problem in order to minimize the total cost of production, storage, and setup.

4. (Perfect matching problem)

We have an even number of  $n$  persons that need to be matched into pairs in order to perform a certain job. If person  $i$  is matched to person  $j$ , there is a cost of  $c_{ij}$ . A matching is a pairing of persons, so that each individual is matched with exactly one other individual. The goal is to find a matching that minimizes the total cost. We represent the set of people by an undirected graph  $G = (N, E)$  where  $N$  is the set of individuals, and the cost of edge  $e = (i, j)$  is  $c_{ij}$ . If  $(i, j) \notin E$ , this indicates that  $i$  and  $j$  cannot be matched. Formulate the perfect matching problem as an integer programming problem. You may use the notation  $\delta(\{i\})$  to denote the set of edges incident to node  $i$ .

5. An airline operates a fleet of 15 jet aircraft, all equipped with the JET32 engine. The airline performs its own engine-related repairs and maintenance at its repair facility. The maintenance director is reviewing the spare parts ordering and stocking policy for the next three years. The JET32 engine consists of 4 main modules, A, B, C, and D. When planes come in for repairs, sometimes the entire engine must be replaced because of extensive damage and wear. More often, however, only certain modules need replacement. The following table 1 contains the forecasted requirements for individual engine modules and complete engines for the next 3 years. The airline places orders for complete engines and modules at the beginning of the year with JET Inc., the manufacturer of the JET32 engine. The following table 2 shows the projected prices for engines and modules the JET Inc. might charge in the next three years.

Note that complete engine cost less than the total cost of buying one module of each type. Assume that the cost of “cannibalizing”, i.e., breaking a complete engine

into four individual modules, is negligible compared to the cost of these modules. The mix of engines and modules the airline orders from JET Inc. must, therefore, account for the economies in ordering complete engines. Assuming that the airline does not have any inventory of modules or engines in hand, formulate an integer programming problem to determine the order quantities for the next 3 years, while minimizing the total cost of purchases. Assume that there are no inventory carrying costs.

Year	Module A	Module B	Module C	Module D	Complete engine
1	5	4	4	2	1
2	2	1	1	7	0
3	3	4	3	0	2

Table 1: Forecasted engine/module requirements.

Year	Module A	Module B	Module C	Module D	Complete engine
1	0.5	2.0	5.0	1.0	7.8
2	0.6	2.2	5.5	1.1	7.5
3	0.7	2.5	6.0	1.3	7.0

Table 2: forecasted engine/module prices.

6. (Assembly line balancing problem)

An assembly line is being designed for manufacturing a discrete part. There are 7 operations numbered 1 to 7 to be performed on each part. Each operation can be started on a part any time after all its immediate predecessor operations given in the following table are completed, but not before. The table also gives the time it takes an operator to carry out each operation, in seconds. The cycle time of the assembly line will be 20 seconds (i.e., each operator will have up to 20 seconds to work on a part before it has to be put back on the line). An operator on the assembly line can be assigned to carry out any subset of operations, as long as the work can be completed within the cycle time and the assignments do not violate the precedence constraints among the operations. It is required to determine the assignments of operations to operators on this line, so as to minimize the number of operators needed.

Formulate this as an integer program. Assume that we know that at most 4 operators are needed to complete the operations.

Operation	Immediate predecessors	Time in seconds
1		7
2		9
3	1, 2	6
4	1	4
5	3	8
6	4	7
7	5, 6	5

(Hint: Consider a dummy operation with processing time of 0, which needs to be completed as the last operation. Then try to minimize the index of the operator (workstation) to which the dummy operation is assigned.)