OR-1 Homework-1

Due: 2018/3/13 (Tue., in class)

1. Consider the two points $x^1 = (-1, 1)$, $x^2 = (1, 0)$ in $\mathcal{R}^2$. Draw the set of points which can be obtained by using the following operations.
   (a) $x = \lambda_1 x^1 + \lambda_2 x^2$, $\lambda_1, \lambda_2 \in \mathcal{R}$ (linear combination)
   (b) $x = \lambda_1 x^1 + \lambda_2 x^2$, $\lambda_1 + \lambda_2 = 1$ (affine combination)
   (c) $x = \lambda_1 x^1 + \lambda_2 x^2$, $\lambda_1, \lambda_2 \geq 0$ (nonnegative linear combination)
   (d) $x = \lambda_1 x^1 + \lambda_2 x^2$, $\lambda_1 + \lambda_2 = 1$, $\lambda_1, \lambda_2 \geq 0$ (convex combination)
2. Let C_i, $i = 1, ..., m$ be convex sets. Show that $\bigcap_{i=1}^m C_i$ is convex.
3. Let $C, D \subseteq \mathcal{R}^n$ be convex sets. Define the set $C + D = \{x + y : x \in C, y \in D\}$. Show that $C + D$ is convex.
4. Let $a_j, j = 1, ..., n$ and b be given scalars. Show that the set $\{x \in \mathcal{R}^n : \sum_{j=1}^n a_j x_j \leq b\}$ is convex.
5. Let $a_{ij}, i = 1, ..., m,\ j = 1, ..., n$ and $b_i, i = 1, ..., m$ be given scalars. Show that the set $P = \{x \in \mathcal{R}^n : \sum_{j=1}^n a_{ij} x_j \leq b_i, i = 1, ..., m\}$ is convex. (Note that P is the set of feasible solutions to a linear programming problem.)
6. Suppose that f_1, f_2 are convex functions from $\mathcal{R}^n$ into $\mathcal{R}$ and let $f(x) = f_1(x) + f_2(x)$. Show that f is a convex function.
7. Suppose that $f_1, ..., f_m$ are convex functions from $\mathcal{R}^n$ into $\mathcal{R}$ and let $f(x) = \sum_{1=1}^m f_i(x)$. Show that f is a convex function.
8. Let $f : \mathcal{R}^n \to \mathcal{R}$ be a convex function. Prove that the function $g(x) = \mu f(x), \mu \geq 0$, $\mu \in \mathcal{R}$ is a convex function.