

## Optimization S/W HW-2

Due : 2018/6/4 (Mon.)

The generalized assignment problem (GAP) is a well-known, NP-complete combinatorial optimization problem that involves finding the minimum cost assignment of  $n$  jobs to  $m$  agents such that each job is assigned to exactly one agent, subject to the agent's available capacity.

Let  $I = \{1, 2, \dots, m\}$  be a set of agents, and let  $J = \{1, 2, \dots, n\}$  be a set of jobs. For  $i \in I$ ,  $j \in J$ , define  $c_{ij}$  as the cost of assigning job  $j$  to agent  $i$ ,  $r_{ij}$  as the resource required by agent  $i$  to perform job  $j$ , and  $b_i$  as the resource available (capacity) of agent  $i$ . Also,  $x_{ij}$  is a 0-1 variable that is 1 if agent  $i$  performs job  $j$ , and 0 otherwise. The mathematical formulation of the GAP is :

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \quad (1)$$

$$\text{subject to} \quad \sum_{i \in I} x_{ij} = 1, \quad \forall j \in J \quad (2)$$

$$\sum_{j \in J} r_{ij} x_{ij} \leq b_i \quad \forall i \in I \quad (3)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in I, \forall j \in J \quad (4)$$

Constraints (2) ensure that each job is assigned to exactly one agent and (3) ensure that the total resource requirement of the jobs assigned to an agent does not exceed the capacity of the agent.

To solve the GAP to optimality, use modeling languages of Xpress-MP or CPLEX. To test the program, use the following data. The format of the given data is given as the following. You should report the optimal integer solution and its optimal value in your homework with the documented program.

**Input format :**

number of agents ( $m$ ), number of jobs ( $n$ )

for each agent  $i$  ( $i=1,\dots,m$ ) in turn:

cost of allocating job  $j$  to agent  $i$  ( $j=1,\dots,n$ )

for each agent  $i$  ( $i=1,\dots,m$ ) in turn:

resource consumed in allocating job  $j$  to agent  $i$  ( $j=1,\dots,n$ )

resource capacity of agent  $i$  ( $i=1,\dots,m$ )

**Data :**

5 20

15 12 16 15 20 18 11 14 23 14 18 13 11 15 17 18 20 11 20 24

22 14 10 15 12 21 24 16 17 22 11 15 18 15 18 23 22 23 21 13

25 11 22 10 22 17 22 12 21 21 16 14 23 21 14 17 25 18 15 15

23 13 14 25 21 18 23 19 17 18 20 20 22 24 23 13 17 22 20 21

15 25 22 14 17 21 16 21 20 12 15 18 17 21 20 20 13 14 23 14

11 11 5 25 10 17 16 20 5 20 14 10 15 6 11 14 13 8 15 11

17 17 12 23 16 25 13 16 19 16 24 17 24 6 6 16 25 21 5 18

13 13 20 17 8 7 16 9 23 18 21 24 11 15 12 24 19 23 13 12

6 10 18 16 12 25 17 7 10 6 20 20 9 20 11 21 6 9 20 23

7 14 18 24 7 24 18 19 18 23 15 16 14 9 16 25 15 20 23 17

42 54 52 47 55