IE531 Linear Programming HW4

(due : 2018/04/30, Mon), in class

1. (a) Let $P = \{x \in \mathcal{R}^n : Ax = b, x \geq 0\}$, where A is an $m \times n$ matrix of rank m. Show that y defines an extreme ray of P (i.e., extreme ray of the recession cone of P) if and only if y is a positive multiple of the vector (x_B, x_N), where $A = \left[B : N\right]$ and B is $m \times m$ invertible, and $x_B = -B^{-1}A_j \geq 0$, x_N is a unit vector with the 1 in the position corresponding to column A_j of N.

   (b) Illustrate part (a) by the following system:

$$\begin{array}{rrrrl} -x_1 & +x_2 & +x_3 & & = 2 \\ -x_1 & +2x_2 & & +x_4 & = 6 \end{array}$$
$$x_j \geq 0 \text{ for all } j.$$

2. Show that a convex cone K has at most one extreme point, namely the origin.

3. Solve the linear program

$$\begin{array}{lrrl} \max & x_1 & +x_2 & \\ & -x_1 & +x_2 & \leq 1 \\ & x_1 & +2x_2 & \leq 4 \\ & & x_1, x_2 \geq 0 & \end{array}$$

   using Fourier-Motzkin elimination.

4. Show that $\{x \in \mathcal{R}^n : Ax = b, x \geq 0\} \neq \emptyset$ is bounded if and only if $Ax = 0$ and $x \geq 0$ imply $x = 0$.

5. Text Chapter 2, #1.