IE 631 Integer Programming HW 4 (due : 2018/ 11/21, Wed.)

Prove the NP-completeness of the following problems. You may use any NP-complete problems discussed in the text or in class notes to prove NP-completeness.

1. Uncapacitated facility location: Given sets M and N and integer c_{ij} for $i \in M, j \in N$, f_j for $j \in N$ and K, is there a set $S \subseteq N$ such that $\sum_{i \in M} \min_{j \in S} c_{ij} + \sum_{j \in S} f_j \leq K$?

2. Set covering: given an $m \times n$ 0-1 matrix A and an integer K, does there exist $x \in B^n$ such that $Ax \geq 1$ and $\sum_{i=1}^{n} x_i \leq K$?

3. Matching with bonds: Given a graph $G = (V, E)$, pairwise disjoint subsets B_i for $i = 1, \ldots, p$ of E, and an integer K, does there exist a matching M in G such that $|M| \geq K$ and, for $i = 1, \ldots, p$, either $B_i \cap M = B_i$ or $B_i \cap M = \emptyset$ (i.e., either all the edges in B_i are in the matching or none are)? The subsets B_i are called bonds.

4. Minimum weight path problem (with positive and negative edge weights): Given a graph $G = (V, E)$, length $l(e) \in Z$ for each $e \in E$, integer K, specified vertices $s, t \in V$. Is there a simple path in G from s to t of length K or less?

5. Given a directed graph $G = (V, A)$, a feedback vertex set is a subset $S \subseteq V$ such that every directed cycle of G contains a vertex in S. Show that the feed back vertex set problem: Does G have a feedback vertex set with $|S| \leq k$, is NP-complete.