IE 631 Integer Programming HW 3 (due : 2018/10/31,Wed.)

1. Text NW: p.109 - 113 , #1, #2, #3, #4, #6

2. Consider the constraint set of a 0-1 knapsack problem $\sum_{j \in N} a_j x_j \le b,\ x \in B^n$. We say that $C \subset N$ is a dependent set if $\sum_{j \in C} a_j > b$. Otherwise C is independent. We also say that a dependent set is minimal if no proper subset of it is dependent. Let C be a minimal dependent set and $E(C) = C \cup \{j \in N:\ a_j \ge \max_{i \in C} a_i\}$. Then $\sum_{j \in C} x_j \le |C| - 1$ (called a cover inequality), and $\sum_{j \in E(C)} x_j \le |C| - 1$ (called an extended cover inequality) are valid inequalities.

a) Show that the cover inequality and the extended cover inequality are valid for the feasible solutions of the knapsack problem.

b) Suppose $E(C) = N$, $C = \{j_1, \dots, j_r\}$ with $a_{j_1} \ge a_{j_2} \ge \dots \ge a_{j_r}$, $a_1 = \max_{j \in N} a_j$, and $(C \setminus \{j_1, j_2\}) \cup \{1\}$ is an independent set. Prove that $\sum_{j \in N} x_j \le |C| - 1$ is a facet.

c) Give an example which shows that $\sum_{j \in E(C)} x_j \le |C| - 1$ is not a facet.

3. Consider a maximum node packing problem on the graph $G = (V, E)$ where $V = \{ 1, 2, ..., 2k+1 \}$, $k \ge 2$ and $E = \{ (1,2), (2,3), ..., (2k+1,1) \}$. Let S be the set of incidence vectors of the feasible node packings in G.

a) What is the dimension of convex hull of S?

b) Show that the odd hole constraint $\sum_{i=1}^{2k+1} x_i \le k$ is a facet defining inequality for the convex hull of S.